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Short-term open-pit production scheduling optimizing multiple objectives accounting for shovel allocation in stockpiles

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Abstract Short-term open-pit mine production scheduling is a challenging task that must deal with several objectives, like maximization of the productivity of equipment (plant and mining), compliance of ore extraction, and others. Unfortunately, there are trade-offs between these objectives that make the problem of finding well-balanced short-term schedules complex to handle. To overcome this problem, we propose a methodology based on mathematical programming and a hierarchical method to generate short-term open-pit schedules considering multiple objectives. The mathematical program allocates shovels to different mining faces, including stockpiles. It considers plant capacity constraints, ore blending, precedences between mining faces, shovels

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throughput, and shovels' traveling time between mining faces. We also propose several compliance indicators, which we use to evaluate and compare different short-term schedules. We apply the proposed optimization model to a real iron open-pit mine to compare how the hierarchical method performs with regards to a single objective approach and show increases of waste extraction compliance up to 29% (when priority is production) and of 73% in production (when the priority is waste extraction). Moreover, in general, we observe that the hierarchical method produces more robust plans.

1 Introduction

Open-pit mining is a technique for the extraction of ore deposits near the Earth's surface that permits access to valuable sections of the deposit with relative ease, therefore reaching high production capacities and lower cost compared to underground mines. However, because the ore distribution is not uniform in the deposit, the excavation process may provide access to parts with various ore content levels (thus, economic value) or even require removing unwanted material ("waste"). Therefore, the order in which different parts of the deposit are accessed has an enormous impact on a mine operation's economic value.

Because of its complexity, the process of planning an open-pit mine is organized into different levels: strategic (long-term), tactical (medium-term), and operational (short-term) levels (L'Heureux et al., 2013). These levels of planning are oriented toward different purposes and designed to work with various levels of information.

Strategic mine planning seeks to maximize the mine operation's economic value, usually associated with the Net Present Value (NPV). For this, it determines what portions of the ore body should be extracted, when this should happen, the total lifespan of the mine, the production rate, and the investment amount. A principal result of a long-term planning is a production schedule, determining the tonnage of waste and ore to be mined from each bench-phase for each year over the scheduling horizon.

Tactical scheduling determines the mining sequence for up to a typical period of 5 years based on the production rate constraints. It takes as input the production schedule determined in the long-term plan and defines what portions will be extracted to meet the production targets with the available resources so that the costs of this extraction are adequately projected and minimized.

Lastly, operational scheduling covers time horizons that may span from a week and up to a year. Within this context, short-term planning seeks to ensure the feasibility of the long and medium-term mine production schedules (Smith, 1998) by delivering the ore tonnes and grades to the processing plant (Chanda, 1990).

To reach long-term goals, short-term planners look to make the best possible use of all available resources: processing facilities, mining zones, stockpiles, and equipment fleet. Unfortunately, there may exist some trade-offs between all these aspects. For example, it may be possible to maximize the productivity of loading equipment by assigning them to mining faces closer to their destination. However, as it turns out, these faces tend to be zones of waste because they are nearer to the surface. Similarly, a mining operation may need blending material from different areas of the mine to obtain certain product quality, thus relocating loading equipment and decreasing its productivity. All this implies that considering only a single criterion for optimization may harm other aspects which are relevant too.

Another relevant aspect of short-term planning is related to stockpiles. Stockpiles are essential because they allow keeping the balance between the ore extracted from the mine and the processing capacity: First, stockpiles act as buffers so that processes before them and processes after them can operate without being constrained by each other. Second, stockpiles can be used to control the blending of material to be processed, reducing its variability. Finally, stockpiles can be used to sort material by grade or other properties (Jupp et al., 2014, Robinson, 2004).

All the issues mentioned above make the problem of short-term mine planning hard to be modeled. In fact, contrary to what happens in long-term and medium-term planning, the short-term planning problem for open-pit mines has not been as widely studied (Blom et al., 2018); therefore, there is a lack of mathematical models and optimization tools to support short-term decisions.

Because of the above, we propose a Mixed-Integer Linear Programming (MILP) problem to generate short-term open-pit schedules. The optimization model's major decision is the assignment, over several time-periods, of shovels to mining faces and stockpiles but accounting for the time required to move from one mining face to another. This integration allows the model to link and balance several of the objectives mentioned before.

The model takes as input production targets (i.e., material to be sent to the mill), mining targets, and waste extraction targets, and looks for a shovel assignment to achieve these targets while also considering the following constraints: plant capacity, ore blending requirements, precedences between mining faces, shovels throughput, mine sequencing and movement of shovels between sectors of the mine.

We utilize the model in two configurations: the single objective approach, which aims to optimize a relevant indicator, and the hierarchical method (Grodzevich and Romanko, 2006) that utilize priorities to sequentially addresses different objectives.

This work's principal contribution is the MILP to generate a short-term schedule for open-pit mines considering loading equipment allocation, stockpiles, and three objectives for optimization: loading equipment productivity, plant productivity, and grade compliance. However, the paper also contributes with several indicators to assess and compare the different short-term schedules. Finally, the article presents the model's application to an iron open-pit mine case study showing the model's validity in different scenarios.

The remainder of this paper is organized as follows. Section 2 provides a review of the related work associated with short-term open-pit scheduling.

Section 3 states the problem being solved by the optimization model. Section 4 presents the proposed optimization model. Section 5 describes the real-scale open-pit mine case study and outlines the scenarios that are analysed. Section 6 reports and discusses the results of the case study. Finally, Section 7 concludes the study and outlines future work.

2 Related work

This section briefly reviews some studies associated with production scheduling at the long- and short-term horizons. We pay special attention to optimization models considering stockpiles and multiple-objectives, which are the core elements of the model proposed in this work.

Most optimization models used to support open-pit planning rely on a discretization of the deposit called the *block model*. A block model is a 3D array of blocks with spatial coordinates and a vector of relevant attributes (tonnage, rock type, ore grades, and others) estimated using samples of the deposit, geologic models, and geostatistical methods. Therefore, optimization models abstract the scheduling process, computing for each block, its extraction period, and its best destination for processing, stocking, or dumping. The primary constraint in open-pit mining is that extraction must happen so that the pit walls do not collapse. This is modelled using *precedence constraints*: For each block, a specific set of blocks (called *predecessors*) must be extracted to gain access to that block while keeping the walls of the pit stable.

The open-pit mine production scheduling problem (OPMPSP) consists of scheduling the blocks' extraction to maximize the Net Present Value (NPV) (Samavati et al., 2018), subject to precedence and capacity constraints. The first formulation of OPMPSP as an integer linear program is due to Johnson (Johnson, 1968). Over the years, several authors have extended this model. The Precedence Constrained Production Scheduling Problem (PCPSP) (Espinoza et al., 2013) is currently regarded as the reference problem for modeling long-term open-pit mining. This model considers several time-periods and multiple potential destinations of the blocks and slope constraints; however, it also considers an arbitrary number of *side* constraints, including capacity and blending restrictions. Unfortunately, neither OPMPSP nor PCPSP consider stocks in their modeling, except as potential block destinations from where no material is retrieved during the planning period.

Indeed, stockpiles are hard to model using linear models because of the mixing of material produced in these piles. As Moreno et al. (2015) states, the inclusion of stockpiles in models for the open-pit production-scheduling problem has been avoided, due to the difficulty of correctly modeling the mixing behavior of the material inside a stockpile. Nevertheless, some authors have proposed various approaches to address this issue.

A few nonlinear optimization models have been proposed to address the open-pit mine production scheduling problem with stockpiles. Tabesh et al. (2015) states that stockpiling should theoretically be modeled nonlinearly to optimize a comprehensive open-pit mine plan, and linearized the formulation by using a "sufficient number" of stockpiles, each with a tight range of grades. Bley et al. (2009) proposed a quadratic model for production scheduling that assumes 'instant-mixing" inside the stockpile, that is, that all grades of material inside the stockpile are averaged. However, this type of model is computationally very difficult to solve, limiting its use in real instances. Bley et al. (2012b) provides more details on computational approaches for solving these models. Bley et al. (2012a) address the solution of the open-pit mine production scheduling problem (OPMPSP) with a single stockpile (OPMPSP+S). The addition of a stockpile adds a relatively small number of quadratic constraints to the formulation of the OPMPSP and turns the problem from a mixed-integer linear into a mixed-integer nonlinear program.

Hoerger et al. (1999) assume that material sent to a stockpile must have a grade within a specific range and that the grade of material extracted from a stockpile is the minimum value of that range. Akaike and Dagdelen (1999) consider that there are infinite potential stockpiles, so every block has its stockpile (i.e., there is no blending in the stockpile). Fu et al. (2019) models the stockpiles using a series of grade bins, allowing the model to allocate material with a different grade. The models of Moreno et al. (2017), Rezakhah and Newman (2020), Rezakhah et al. (2020) utilize blending constraints to require a constant grade in the stockpiles by forcing that the average grade sent to each stockpile is known and fixed in advance. Moreno et al. (2017) proposed several linear integer optimization problems to schedule open-pit mines considering stockpiling. Rezakhah et al. (2020) apply those models to an operational poly-metallic (gold and copper) mine, where stockpiles are used to blend materials based on multiple block characteristics. Rezakhah and Newman (2020) extend the model to consider the deterioration of material when exposed to the environment.

Asad (2005) presents a long-term cut-off grade optimization algorithm for open-pit mining operations with stockpiling in a deposit with two economic minerals. This algorithm is an extension of the theory of cut-off grades in deposits of two economic minerals presented in Lane et al. (1984).

Finally, some articles account for stockpiles in the context of geological uncertainty in the metal content, i.e., they incorporate the fact that geostatistical methods produce only estimations of critical values like grades Koushavand et al. (2014), Lamghari and Dimitrakopoulos (2016), Levinson and Dimitrakopoulos (2019), Ramazan and Dimitrakopoulos (2013), Silva et al. (2015).

The works mentioned above addressed block scheduling in a long-term setting; therefore, they are not concerned with operational aspects like equipment assignment, which is central to the problem we address. We now review some works oriented to the short-term.

Fioroni et al. (2008) seeks to reduce mine costs using optimization and simulation models to generate short-term production schedules that can be executed in the mine, considering the actual use of mining equipment.

Rehman and Asad (2010) propose a MILP model to define the short-term sequence mining of blocks of a limestone quarry to meet plant quantity and quality requirements at the lowest possible cost. Contrary to our approach, their model does not consider shovel allocation to quarry blocks.

Eivazy and Askari-Nasab (2012) develop a MILP model to generate short-term schedules. The model minimizes the total cost, considering: mining cost, processing cost, waste rehabilitation cost, rehandling cost and total haulage cost. The model takes into account multiple destinations and models: stockpiles as buffers and blending location, horizontal directional mining, and decisions on-ramps, blending constraints at the processing plants, mining and processing capacities, and mining precedences. The model assumes that stockpiles' output grade is equal to their average grade; however, it does not consider the allocation of loading equipment in the mine or to stockpiles.

Torkamani and Askari-Nassab (2015) integrates the optimal scheduling of short-term production with the simulation of the shovel-truck operation; based on this, it is possible to choose the optimal number of shovels and trucks needed to fulfill the mine schedule.

Upadhyay and Askari-Nasab (2016, 2018, 2019) present an MILP problem to allocate shovels to mine faces to maximize production, achieve the desired head grade and tonnage at crushers, and minimize shovel movements.

Mousavi et al. (2016) propose a MILP problem that considers precedences, machine capacity, grade requirements, processing demands, and stockpile management. The objective function is to minimize the total cost, including the cost of rehandling, holding, misclassification, and drop-cut. Their model considers the assignment of loading equipment in the mine but not to stockpiles.

Matamoros and Dimitrakopoulos (2016) propose a formulation based on stochastic mixed-integer programming to address scheduling of open-pit mines in the short-term. The model considers uncertainty in both ore body metal quantity and quality. It also takes into account fleet parameters and equipment availability. The model allocates shovels to mine sectors and the number of truck trips per shovel. The objective function considers operating fleet cost and mining cost. Stocks are modeled implicitly by considering a penalization cost due to overproduction at each time-period.

Blom et al. (2017) apply the hierarchical method to generate multiple, diverse short-term schedules while optimizing for a customizable, prioritized sequence of objectives. They use a rolling horizon-based algorithm to resolve instances.

Upadhyay and Askari-Nasab (2019) uses a mixed-integer linear objectives programming model (MILGP) to obtain the optimal allocation of shovels and trucks to meet the objectives aligned to long-term scheduling: maximize production, minimize deviations of the head grade, minimize deviations of the plant feed tonnage, and minimize operating costs.

Alexandre et al. (2019) deals with the truck dispatching problem. That is, the efficient allocation of trucks to shovels in operation at open-pit mines. The work present multi-objective strategies for solving the problem of dynamically allocating a heterogeneous fleet of trucks in an open-pit mining operation.

Both and Dimitrakopoulos (2020) presents a stochastic optimization model that optimizes the short-term extraction sequence of an open-pit mine allocating shovels and trucks to mine faces. The model considers geological and equipment performance uncertainty.

Shah and Rehman (2020) describes a mixed integer linear programming model that optimizes the short-term extraction sequence of an cement quarry allocating shovels and trucks to mine faces. The objective function minimizes the truck and shovel total cost, subject to quantity and quality constraints.

Benlaajili et al. (2020) presents a tuck-shovel dispatching model that allocates shovels to mine faces and shovels to shovels in two main steps. The first step proposes a modeling of the allocation of shovels problem as a vehicle routing problem. In the second step, a mixed integer linear programming model is proposed to determine the optimal number of trips required to transport the quantity of ore from each loading point to each dumping site, this model is used to dispatch available trucks to the appropriate shovel.

Table 1 summarizes and compares the characteristics of the short-term open-pit mine scheduling articles that we have reviewed so far.

With regard to works in Table 1 that consider multiples objectives, Fioroni et al. (2008) considers the following objectives: maximizing ore production, minimizing grade deviation to ore plant, and minimizing the loss of production due to load equipment movements. Torkamani and Askari-Nasab (2011) minimizes the operational cost associated with the mine considering: cost of moving shovels to new faces, total transportation cost of trucks moving to the waste dump or to the mill, and cost of negative deviation from the production target at the mill. Eivazy and Askari-Nasab (2012) also minimizes the

Feature / Article	Fioroni et al. (2008)	Torkamani and Askari-Nasab (2011)	Eivazy and Askari-Nasab (2012)	Mousavi et al. (2016)	Matamoros and Dimitrakopoulos (2016)	Blom et al. (2017)	Upadhyay and Askari-Nasab (2019)	Alexandre et al. (2019)	Both and Dimitrakopoulos (2020)	Shah and Rehman (2020)	Benlaajili et al. (2020)
Shovel allocation in mine faces	✓	✓	✗	✓	✓	✓	✓	✗	✓	✓	✓
Shovel allocation in stockpiles	✗	✗	✗	✗	✗	✓	✗	✗	✗	✗	✗
Explicit modeling of stockpiles	✗	✗	✓	✓	✗	✓	✗	✗	✓	✗	✗
Truck allocation in mine faces	✓	✓	✗	✗	✓	✓	✓	✓	✗	✓	✓
Grade uncertainty	✗	✗	✗	✗	✓	✗	✗	✗	✓	✗	✗
Equipment performance uncertainty	✗	✗	✗	✗	✓	✗	✗	✗	✓	✗	✗
Multi-objective optimization (not only costs)	✓	✗	✗	✗	✓	✓	✓	✓	✓	✗	✓

Table 1: Comparison of short-term open-pit mine scheduling articles.

total cost, considering: mining cost, processing cost, waste rehabilitation cost, rehandling cost and total haulage cost. Mousavi et al. (2016) minimizes the total cost, which includes rehandling and holding costs, misclassification and drop-cut costs. The misclassification cost is monitored to ensure that material is assigned to the right destination. A drop-cut is a condition such that a block is extracted while all the adjacent blocks have not yet been extracted. In contrast to a drop-cut, a side-cut is performed when the excavator is located in the same bench as the block. Finally, a drop-cut cost is considered in order to give priority to the side-cut extraction, unless a new working bench is required to be opened. The optimization problem of Matamoros and Dimitrakopoulos (2016) minimizes the overall extraction cost. This cost considers the following components: cost of extracting material from the mine, hauling cost given the uncertainty in the trucks' hauling time and mechanical availability, cost of shovel movements among sectors, lack of production per shovel given uncertainty in its mechanical availability, penalize the lack of mining blocks that match the required mining width and minimize the geological risk with respect to the quality and quantity of ore production and penalize deviation from production targets. Blom et al. (2017)´s formulation considers multiple objectives such as maintaining alignment with a longer-term plan and maintaining product grades within desired bounds. The optimization problem described by Upadhyay and Askari-Nasab (2019) considers the following objectives: maximization of production, meeting the desired feed to processing plants, meeting the grade blending requirements of the processing plants, and minimizing shovel movements. The objective function for the truck allocation described by Alexandre et al. (2019) is to maximize production and minimize costs. The optimization model proposed by Both and Dimitrakopoulos (2020) maximizes metal production and profit of the mining complex as a whole, instead of minimizing operational costs. It considers: revenue and costs in the mining complex, penalties for deviations from production targets, reduce the risk of not achieving shovel production targets per mining area, reduce risk of falling short of truck haulage capacity, shovel movement cost, account for the cost of trucks in operation, and smoothing of the mining schedule. For this part, the objective function of the optimization problem described in Shah and Rehman (2020) considers the minimization of truck/shovel cost. Finally, Benlaajili et al. (2020) addresses two problems. The first problem (the allocation of shovel to mining faces) minimizes the total travel cost of shovels. The second problem (allocation of trucks to shovels) minimizes the number of trips required to transport ore during a working shift.

From Table 1 we observe that the only two works that consider uncertainty are Matamoros and Dimitrakopoulos (2016) and Both and Dimitrakopoulos (2020). These articles take into account geological uncertainty and the equipment performance uncertainty. We understand that addressing the mentioned uncertainties is critical to risk assessment and decision making in short-term open-pit mine production scheduling; however, we consider stochastic modeling out of the scope of this study and a potential improvement for future research.

2.1 Multi-objective optimization

There are several different multi-objective evolutionary algorithms like the genetic algorithm which fundamentally operates on a set of candidate solutions. The weighted sum and hierarchical method are commonly used to optimize multiple objectives.

In the weighted sum method, each different objective is assigned a positive coefficient that represents its relative importance. Then, a unified objective function is constructed as the addition of the objectives multiplied by their corresponding weights.

In the hierarchical method, the decision-maker ranks the objective functions, sorting the objectives in descending order of importance. In this method, as many optimization problems as objective functions are solved sequentially, following the rank previously defined. Each problem is then solved for its corresponding objective, but with an additional constraint requiring it to perform as well as the optimal solutions of the criteria already considered.

Figure 1 provides a schematic view of this procedure for N objective functions to be minimized in descending order of importance. The j-th objective function is denoted as A_j, and $\overline{A}_j$ as the optimal value of this function in the j-th problem. The set of general constraints of the problem are represented as $x \in S$, where x is the vector of variables and S the set of feasible points. The parameters $\lambda_i \geq 1, \forall i \in \{1, ..., N\}$ represent the tolerance of the deviation of the value $\overline{A}_i$. Notice that if $\lambda_i = 1$, it means that the $i+1$-th sub-problem must find a solution that replicates the i-th optimum value obtained for the previous sub-problem, while $\lambda > 1$ implies that some deterioration of the i-th objective value is allowed.

At the first iteration, the first problem is solved and so a solution x_1 with value $\overline{A}_1 = A_1(x_1)$ is found. Then, for the j-th iteration, solutions $x_k, k = 1, \ldots, j-1$ have been found, each with a value $\overline{A}_k = A_k(x_k)$, hence the constraints $A_k(x) \leq \lambda_k \overline{A}_k$ are added for $k = 1, \ldots, j-1$ and the problem with

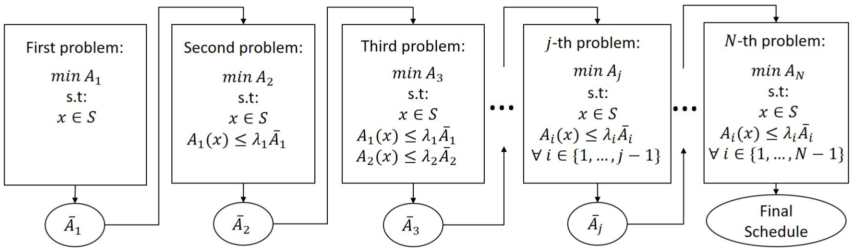

Fig. 1 Scheme of the hierarchical method considering N objectives.

objective function A_j is solved. The process continues until the last problem, with $j = N$ is solved and its solution x_N and its value $A_N(x_N)$ are reported.

The hierarchical method seems more suited for short-term scheduling and therefore is the one utilized in this article. In general, the articles that consider multiple objectives described in Table 1 use the weighted sum method to perform multi-objective optimization. Applying the weighted sum method is straightforward and easier to understand than the hierarchical method. However, it requires selecting the weight coefficients, which makes its application difficult (Grodzevich and Romanko, 2006). An advantage to use the hierarchical method to perform a multi-objective optimization is that we do not have to compute or select the value of the weight coefficients. As a drawback, we need to solve n optimization problem to perform a multi-objective optimization with n hierarchical objectives.

3 Problem statement

This section provides an overview of all the modeling aspects incorporated in the mathematical model. The precise optimization problem, notation and parameters are introduced in Section 4.

Each mining face is characterized by a tonnage, type of material (ore or waste), and the grade of the metal of interest. We assume two kinds of faces: mine or stockpiles. Mining faces only allow extraction, while stockpile faces also allow the accumulation of material extracted at the mine. Mining faces are also related to each other by precedences, meaning that some need to be depleted before others' extraction begins.

The material at the mining faces may contain different components (for example, different metals or pollutants). We will assume that this composition is known and there are targets or ranges allowed at the plant, for each period and component.

In short-term planning, the traveling time of shovels can be a significant portion of the total time; thus, we consider the travel time required to move a shovel between two different faces as not available for production.

In our approach, stockpiles are potential destinations of material extracted at the mine and locations to which shovels can be assigned. In terms of the grades, we adopt the same model as (Hoerger et al., 1999) and assume that the stockpiles are homogeneous and the ore reclaimed from each stockpile has a grade equivalent to the average grade of the stockpile, which is constant over the scheduled horizon. Moreover, we assume that the stockpile to which the face sends its material is defined in advance, i.e., the model decides the amount to be sent but not what stockpile to use.

We consider that a shovel is *unavailable* during meals, shift changes, maintenance, and failures and assume that the total unavailable time per period is known in advance for every shovel; thus, the model can use a shovel only during its available time. Notice that the model does not utilize the internal distribution of unavailable time explicitly. In particular, no maintenance or failures are incorporated directly and the model can use only available time.

The available time of a shovel during a period can be split as follows: $AvailableTime = TravelingTime + OperationTime + StandByTime$. Traveling occurs when the shovel moves from one mining face to another during a certain planning period. Operational time happens when the shovel is loading material and sending it to its destination. Stand-By time occurs only in the case when the face is depleted, that is if the operational time is more than enough to load all the material.

To compute the material loaded, we consider that shovels have a known operative throughput that quantifies the material that they can load per unit of time (tonnes per hour). We assume that this parameter depends on the shovel and the mining face. This is to allow the model to consider different shovels, but also different conditions that may effect the performance of a shovel, like limited space for operating the equipment, delays due to long hauling distances, and others.

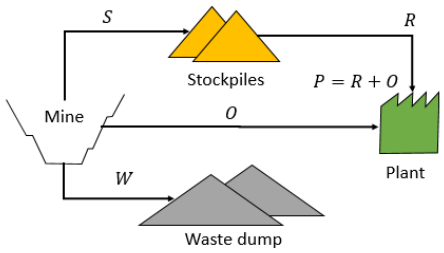

Fig. 2 Conceptual diagram of a material's flow network in a mining operation.

We assume that the time (in hours) that is required by a given shovel to move between two mining faces is also known in advance. However, as a shovel may visit more than one face per time-period, we utilize *routes*. A route is a list of sectors in which a shovel travels sequentially within a time-slot. Thus, the traveling time is computed as the addition of all movements between mining faces in the route.

We propose several indicators to compare and evaluate the performance of the results beyond the optimization targets. These indicators aim to measure the compliance of the plan, i.e., how close the material flows scheduled by the plan are to their targets. The indicators are based on the material flows depicted in Figure 2: O, the tonnage of ore sent directly from the mine to the processing facilities; R the tonnage of rehandling, that is, material sent from stockpiles to the processing facilities; S, the tonnage of material sent from the mine to stockpiles; and W, the tonnage of material sent to waste dumps.

Two critical material flows set by long-term scheduling are: $P = R + O$, the total material sent to the plant, and $M = S + O + W$, the total material extracted from the mine. Therefore, we consider total targets P^0 and M^0, respectively, to be moved during the planning horizon. We consider also W^0, a waste removal target. It is worth noting that, W^0 is not usually considered in long-term planning, where all ore extraction occurs during the length of time being scheduled; however, in short-term planning it is a critical target, because it enables reaching ore in periods beyond the planning horizon.

Based on the material flow targets described before, we introduce *compliance performance indicators*, which are presented in Table 2: waste extraction, plant utilization, plant utilization due to ore extracted from the mine, and ore extraction. These indicators will be later used to analyze the plans obtained using the model.

3.1 Multiple criteria for measuring short-term planning performance

We consider several potential criteria (and therefore objective functions) to be minimized using the model. In this article, we focus on the first three of the list below, although the model could be used also to analyze the others:

- ΔO is the deviation (in tonnage) between ore sent to the plant from the mine and the plant capacity.
- ΔP is the deviation (in tonnage) between ore sent to the plant from the mine and stockpiles and the plant capacity.
- ΔW is the deviation (in tonnage) between waste hauled and its target.
- ΔD is the deviation (in tonnage) between ore sent to the plant and the ore plant capacity per period.

Index	Formula	Description
Γ_W	$\frac{W}{W^0} \cdot 100$	Waste extraction compliance.
Γ_P	$\frac{O+R}{P^0} \cdot 100$	Plant utilization.
Γ_O	$\frac{O}{P^0} \cdot 100$	Plant utilization due to ore directly sent from mine.
Γ_M	$\frac{O+S}{M^0} \cdot 100$	Ore extraction compliance

Table 2: Compliance Performance indicators.

- $\Delta Grade_j$, the deviation (in tonnage) between the content of component j sent to the plant and its target.
- $TotalTravelTime$, the total traveling time of the shovel fleet (in hours).
- $TotalTravelCost$, the total traveling cost of the shovel fleet (in dollars).

4 Optimization model

This section presents in detail the mathematical model that implements the problem described in the previous section as a mixed integer linear program. The main decision made by the model is the allocation of loading equipment to mining faces, which is done considering the fraction of a time-slot that the shovel will be assigned and the fraction of time that it will be working in a mining face.

4.1 Sets and Indexes

- $p \in \mathcal{P}$, the set and index for shovels.

- $f, f' \in \mathcal{F}$, the set and indexes for mining faces.
- $\mathcal{F}^{ore}, \mathcal{F}^{waste} \subset \mathcal{F}$, sets of mining faces that contain ore and waste, respectively.
- $\mathcal{F}^{sp} \subset \mathcal{F}$ set of mining faces that correspond to stockpiles. (Notice that $\mathcal{F}^{sp} \subset \mathcal{F}^{ore}$, because all stockpiles contain ore).
- $t \in \mathcal{T} = \{1, 2, \ldots, T\}$, the set and index for time periods. T is the time horizon.
- $j \in \mathcal{J}$, the set and index for material components to be controlled (like grades, pollutants or others).
- $r \in \mathcal{R}$, the set of routes. A route r is a ordered sequence of mining faces $r = (f_1, f_2, \ldots, f_k)$, for some k, which corresponds to the length of the route, denoted as $|r|$.
- $\mathcal{R}_f$, the set of routes that go through mining face f.
- $\mathcal{S}_f \subset \mathcal{F}, f \in \mathcal{F}^{sp}$ is the set of faces that send material to stockpile f.
- $\mathcal{H}_r$, the set of routes whose last face is equal to the first face of route r.
- $(f, f') \in \mathcal{Q}$, the set of precedences between the mining faces (f' has to be mined before f).

4.2 Parameters

The parameters of the optimization model are detailed below.

- $Ton_f \geq 0$, the total material to be mined in mining face f (in tonnes).
- $Grade_{j,f} \in [0, 1]$, the fraction of the content j in the mining face f.
- $Length_t > 0$, the length of period t (in hours).

- $AvlTime_{p,t}$, hours that shovel p is available during period t. This time is computed considering the expected maintenance time $MaintTime_{p,t}$, failure time $FailTime_{p,t}$, and the delays $DelayTime_{p,t}$ as follows:

$$AvlTime_{p,t} = Length_t - (MaintTime_{p,t} + FailTime_{p,t} + DelayTime_{p,t}).$$

- $Prod_{p,f}$, the tonnage of material that can be mined by the shovel p in the mining face f. If $\theta_{p,f}$ is the estimated throughput of shovel p at face f (in tonnes per hour) then

$$Prod_{p,f} = AvlTime_{p,t} \cdot \theta_{p,f}.$$

- $Cap_t > 0$, the processing capacity of the plant in period t (in tonnes).
- $MinProd_t > 0$, the minimum desired tonnage to be sent to the ore processing plant in period t (in tonnes).
- $MaxGrade_j \geq 0, j \in \mathcal{J}$, the maximum grade of the content j to be sent to the ore processing plant in period t.
- $MinGrade_j \geq 0, j \in \mathcal{J}$, the minimum grade of content j to be sent to the ore processing plant in period t.
- $TravelTime_{p,r} > 0, p \in \mathcal{P}, r \in \mathcal{R}$ is traveling time of shovel p along the sectors of route r (in hours).
- $MaxMoves_p, p \in \mathcal{P}$, the maximum number of movements between sector of shovel p over the scheduling horizon.
- $TargetGrade_j \geq 0, j \in \mathcal{J}$, the grade target of the content j by the ore processing plant (in percentage).
- N, a very large number.

4.3 Variables

To ease the reading of the model, all binary variables have a bar on top of it, while continuous variables do not have the bar.

The decision variables are used to determine the location and duration of shovel assignments, to control the movement of the shovels, to determine what mining faces are currently active, and to account for tonnages:

- $x_{p,f,t} \in [0,1]$, fraction of period $t \in \mathcal{T}$ that shovel $p \in \mathcal{P}$ is assigned to face $f \in \mathcal{F}$.
- $\bar{x}_{p,f,t} \in \{0,1\}$, equal to 1 if shovel $p \in \mathcal{P}$ is allocated to mining face $f \in \mathcal{F}$ in period $t \in \mathcal{T}$, 0 otherwise.
- $y_{p,f,t} \in [0,1]$, fraction of the period $t \in \mathcal{T}$ where shovel $p \in \mathcal{P}$ is operational at mining face $f \in \mathcal{F}$, i.e., the shovel is loading and sending material to its destination.
- $z_{p,f,t} \in [0,1]$, the time percentage of the period $t \in \mathcal{T}$ where shovel $p \in P$ is operative at mining face $f \in \mathcal{F}$ sending material to its predefined stockpile.
- $\bar{w}_{f,t} \in \{0,1\}$, equals to 1 if mining face $f \in \mathcal{F}$ finishes its exploitation in period $t \in \mathcal{T}$ or before, 0 otherwise.
- $\bar{v}_{p,r,t} \in \{0,1\}$, equals to 1 if shovel $p \in \mathcal{P}$ goes through route $r \in \mathcal{R}$ in period $t \in \mathcal{T}$, 0 if not.
- $\beta_{f,t} \geq 0$, tonnage of the stockpile $f \in \mathcal{F}^{sp}$ at the end of the period $t \in \{0\} \cup \mathcal{T}$. (For simplicity, in the model, $\beta_{f,t=0}$ is fixed to the initial tonnage in stockpile f.)

Thus, the optimization model decides if the material contained in mining face of ore f that is not a stockpile is either sent directly to the plant or sent to the predefined stockpile.

4.4 Objective functions

We consider different possible objective functions, which are listed below. These objective functions correspond to deviations from targets like production, desired grades, or waste removal.

The deviations that we consider were introduced in Section 3.1.The explicit mathematical expressions of these quantities are presented in Equations (1)-(8). (Notice that ΔD is defined in terms of Δd_t.)

$$\Delta O = M^0 - \sum_{p\in\mathcal{P}, f\in\mathcal{F}^{ore}\setminus\mathcal{F}^{sp}, t\in\mathcal{T}} Prod_{p,f} \cdot y_{p,f,t} \tag{1}$$

$$\Delta P = P^0 - \sum_{p\in\mathcal{P}, f\in\mathcal{F}^{ore}, t\in\mathcal{T}} Prod_{p,f} \cdot y_{p,f,t} \tag{2}$$

$$\Delta W = W^0 - \sum_{p\in\mathcal{P}, f\in\mathcal{F}^{waste}, t\in\mathcal{T}} Prod_{p,f} \cdot y_{p,f,t} \tag{3}$$

$$\Delta d_t = Cap_t - \sum_{p\in\mathcal{P}, f\in\mathcal{F}^{ore}} Prod_{p,f} \cdot y_{p,f,t} \qquad \forall t \in \mathcal{T} \tag{4}$$

$$0 \leq \Delta d_t \leq \Delta D \qquad \forall t \in \mathcal{T} \tag{5}$$

$$\sum_{p\in\mathcal{P}, f\in\mathcal{F}^{ore}} Prod_{p,f} \cdot Grade_{j,f} \cdot y_{p,f,t} + \Delta Grade^-_{j,t} - \Delta Grade^+_{j,t} = TargetGrade_j \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{6}$$

$$\Delta Grade^+_{j,t} \leq \Delta Grade_j \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{7}$$

$$\Delta Grade^-_{j,t} \leq \Delta Grade_j \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{8}$$

4.5 Constraints

Now, we present the mathematical expressions corresponding to the constraints of the optimization model. Second, we describe them in groups associated to different concepts. Due to the complexity of the model, we group the constraints and provide separate descriptions.

4.5.1 Production constraints

Constraints (9)-(12) impose the ore plant capacity and models the distribution of time of each shovel.

Constraint (9) imposes that the total material extracted in each mining face along the planning horizon must be less or equal than the total material contained in that mining face. Constraint (10) sets the minimum and maximum ore tonnages sent to the ore processing plant. Constraint (11) limits the minimum and maximum contents of component j sent to the ore processing plant. Constraint (12) models the shovel time by imposing that the effective shovel time plus the shovel movement time between sectors are less than or equal to the maximum shovel time utilization.

$$\sum_{p \in \mathcal{P}, t \in \mathcal{T}} Prod_{p,f} \cdot y_{p,f,t} \leq Ton_f \qquad \forall f \in \mathcal{F} \tag{9}$$

$$MinProd_t \leq \sum_{p \in \mathcal{P}, f \in \mathcal{F}} Prod_{p,f} \cdot y_{p,f,t} \leq Cap_t \qquad \forall t \in \mathcal{T} \tag{10}$$

$$MinGrade_j \cdot \sum_{p\in\mathcal{P},f\in\mathcal{F}^{ore}} Prod_{p,f} \cdot y_{p,f,t} \leq \sum_{p\in\mathcal{P},f\in\mathcal{F}^{ore}} Grade_{j,f} \cdot Prod_{p,f} \cdot y_{p,f,t} \leq MaxGrade_j \cdot \sum_{p\in\mathcal{P},f\in\mathcal{F}^{ore}} Prod_{p,f} \cdot y_{p,f,t} \qquad \forall t \in \mathcal{T}, j \in \mathcal{J} \quad (11)$$

$$\sum_{f\in\mathcal{F}} Length_t \cdot x_{p,f,t} + \sum_{r\in\mathcal{R}} TravelTime_{p,r} \cdot \bar{v}_{p,r,t} \leq AvlTime_{p,t} \quad \forall p \in \mathcal{P}, t \in \mathcal{T} \quad (12)$$

4.5.2 Shovel allocation constraints

The dynamics of shovel assignment to mining faces is determined by constraints (13)-(17): Constraint (13) sets the precedences between faces, not allowing a shovel to work in a face before the previous face is completely extracted. Constraint (14) imposes that a shovel can allocate time for working in face f during period t only if it has been assigned to that f during t. Constraint (15) ensures that when a mining face is depleted, no other shovel can be assigned to it (this prevents shovels from using "old" mining faces as shortcuts to move between different places in the mine). Constraint (16) imposes that extraction of mining face f is not fulfilled in period t until all the material in the face is completely extracted. Constraint (17) ensures that if extraction of mining face f is concluded in a period t, then it remains in that state until the end of the scheduling horizon.

$$x_{p,f,t} \leq \bar{w}_{f't} \qquad \forall (f, f') \in \mathcal{Q} \quad (13)$$

$$x_{p,f,t} \leq \bar{x}_{p,f,t} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \quad (14)$$

$$\bar{x}_{p,f,t} \leq 1 - \bar{w}_{f,t-1} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \setminus \{1\} \tag{15}$$

$$\bar{w}_{f,t} \leq \sum_{p \in P, t \in \mathcal{T}} \frac{Prod_{p,f}}{Ton_f} \cdot y_{p,f,t} \qquad \forall f \in \mathcal{F}, t \in \mathcal{T} \tag{16}$$

$$\bar{w}_{f,t} \geq \bar{w}_{f,t-1} \qquad \forall f \in \mathcal{F}, t \in \mathcal{T} \setminus \{1\} \tag{17}$$

4.5.3 Shovel movement

Constraints (18)-(21) model the movement of each shovel between mine faces of different sectors.

Constraint (18) sets the maximum number of movements between mining faces along the scheduled horizon for each shovel. Constraint (19) ensures that shovel p uses at most one route during period t. Constraint (20) makes sure that, during period t, shovel is allocated only to mining faces that are visited during that period. Constraint (21) ensures that the movement between different faces is consistent, i.e., that the last mining face of the route r and the first mining face of the r' route must be equal.

$$\sum_{r \in \mathcal{R}, t \in \mathcal{T}} (|r| - 1) \cdot \bar{v}_{p,r,t} \leq MaxMoves_p \qquad \forall p \in \mathcal{P} \tag{18}$$

$$\sum_{r \in \mathcal{R}} \bar{v}_{p,r,t} \leq 1 \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{19}$$

$$\bar{x}_{p,f,t} \leq \sum_{r \in \mathcal{R}_f} \bar{v}_{p,r,t} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \tag{20}$$

$$\bar{v}_{p,r,t} \leq \sum_{r' \in \mathcal{H}_r} \bar{v}_{p,r',t-1} \qquad \forall p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T} \setminus \{1\} \tag{21}$$

4.5.4 Stockpile constraints

Constraints (22)-(25) models the stockpiles.

Constraint (22) defines the time allocated to send material to stockpiles. Constraint (23) imposes that stockpiles do no sent material to stockpiles. Constraint (24) sets the initial ore tonnage of the mining faces that are of type stockpile. Constraint (25) defines the inventory of ore tonnage in a stockpile.

$$y_{p,f,t} + z_{p,f,t} \leq x_{p,f,t} \qquad \forall p \in \mathcal{P}, \in \mathcal{F}, t \in \mathcal{T} \tag{22}$$

$$z_{p,f,t} = 0 \qquad \forall p \in \mathcal{P}, \in \mathcal{F}^{sp}, t \in \mathcal{T} \tag{23}$$

$$\beta_{f,0} = Ton_f \qquad \forall f \in \mathcal{F}^{sp} \tag{24}$$

$$\sum_{p \in \mathcal{P}, f' \in \mathcal{S}_f} Prod_{p,f'} \cdot z_{p,f',t} + \beta_{f,t-1} = \sum_{p \in \mathcal{P}} Prod_{p,f} \cdot y_{p,f,t} + \beta_{f,t} \qquad \forall f \in \mathcal{F}^{sp}, t \in \mathcal{T} \tag{25}$$

5 Case study

In this section, we describe the real-scale open-pit mine case study and outline the different experiments that were performed.

5.1 Mine operation description

The case study comprises mining faces distributed in six sectors, so the notation for mining face names is `N_LEVEL`, where `N` is the sector and `LEVEL` is the z coordinate. The mine planning horizon is one month, split into ten periods, each lasting between one and four days. The main parameters related to the mine operation are summarized in Tables 3 to 7: Table 3 summarizes the processing capacities and period lengths for each period ($Prod_t = 0, \forall t$, i.e., there is no minimum production requirement). Table 4 shows the parameters related to shovels. Table 5 presents the tonnage of ore and waste contained in each mining face. In this case study, mining face 6_1380 is the only stockpile and has 0 tons of inventory at the beginning of the scheduling horizon. Table 6 presents the targets for production and material flow defined by the long-term plan. Finally, Table 7 reports the traveling distance between sectors (travel time for mining faces in the same sector is assumed to be negligible).

5.2 Experimental design

The model is used under different configurations to analyze how the performance indicators presented in Section 3.1 change under different circumstances. There are a total of 28 configurations, which are obtained by considering the alternatives described below. The remaining constraints and parameters are common to all instances.

- **Optimization criteria.** In this case the following deviations are considered as objectives for minimization (a) plant utilization (ΔP), (b) waste extraction (ΔW), and (c) plant utilization due to ore directly sent from mine (ΔO) (see Section 4). Each of these criteria is evaluated using a single-step optimization and several two steps hierarchical configurations. Table 8

Period	Period length [days]	Ore plant capacity [kt]
1	1	0
2	4	42
3	3	0
4	4	42
5	3	0
6	4	42
7	3	0
8	4	39
9	3	0
10	2	15
Total	31	180

Table 3: Length of periods and ore plant capacity.

Shovel	Effective throughput [t/h]	Maximum utilization [%]	Speed [km/h]
1	1,350	43	2
2	1,300	35	2
3	1,200	47	15
4	1,150	63	7
5	1,150	45	7
6	1,200	40	15

Table 4: Shovel fleet parameters.

Mining face	Sector	Waste [kt]	Ore [kt]
1_ 1380	1	335	4
2_1330	2	40	2
2_1320	2	299	41
3_1280	3	21	2
3_1270	3	102	20
3_1260	3	131	10
3_1250	3	22	2
3_1240	3	618	94
4_1230	4	85	4
5_1240	5	59	0
5_1230	5	288	1
6_1380	6	0	0
Total tonnage		$1,999$	180

Table 5: Ore and waste tonnage of mining faces (in [kt]).

Indicator	Value [kt]	Description
P^0	180	Total Ore sent to the plant.
W^0	1,999	Total waste movement.
M^0	180	Total extracted material from the mine.

Table 6: Material flow targets for the planning horizon.

Sectors	1	2	3	4	5	6
1	-	1.7	2.4	2.8	3.2	2.4
2	1.7	-	0.7	1	1.5	1.6
3	2.4	0.7	-	0.4	0.5	1.3
4	2.8	1	0.4	-	0.1	1.8
5	3.2	1.5	0.5	0.1	-	2.1
6	2.4	1.6	1.3	1.8	2.1	-

Table 7: Distance between sectors (in km)

summarizes the criteria considered for the single optimization and hierarchical optimization in each case. For example, configuration $\Delta P(\Delta W)$ corresponds to the case where ΔP is minimized subject to ΔW, i.e., first the waste deviation ΔW is minimized and then plant deviation ΔP is minimized, subject to the waste deviation, hierarchically. Finally, in terms of the tolerance parameters used in the hierarchical method, we set $\lambda_i = 1$, that is, we do not permit any deterioration in the values when compared with a single-objective approach. The main reason for this is that the focus of this work is comparing how different configurations (objective functions and hierarchies) impact the results in terms of their performance, which is difficult to be done objectively if the λ parameters are different for each configuration.

– **Presence or absence of the stockpile.** These two options are labeled as "Yes" and "No", respectively, in the corresponding results.

Opt. criteria	Notation	Description of target to be Minimized
Single	ΔW	Waste deviation from target.
	ΔP	Total ore sent to the plant deviation from production target.
	ΔO	Deviation of ore sent directly from the mine to the plant.
Hierarchical	$\Delta P(\Delta W)$	Plant deviation s.t. minimum waste deviation.
	$\Delta W(\Delta O)$	Waste deviation s.t. minimum ore deviation.
	$\Delta W(\Delta P)$	Waste deviation s.t. minimum plant deviation.
	$\Delta O(\Delta W)$	Total ore deviation s.t. minimum waste deviation.

Table 8: Different optimization of short-term objectives considered in case study.

– **Fixed or mobile shovel fleet.** For this, two possible configurations are considered: a *static* fleet, meaning that shovels must remain in the same sector where they start, and fully *mobile* fleet, in which case shovels could change sector at most once during the planning horizon.

5.3 Computational resources

All the schedules presented in this study were obtained on a 2.60 GHz Intel® Xeon® CPU, with 256 GB RAM, running Windows 8®. The optimization model was solved using Gurobi Optimizer version 8.1 (Gurobi Optimization, 2019). We impose a minimum MIP gap of 5.0% when solving the optimization problems. We believe that this value is an adequate trade-off between the

Result	Running Time [min]				MIP gap [%]				Objective Value			
Fleet	Static		Mobile		Static		Mobile		Static		Mobile	
Stockpile	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
ΔW	0.5	0.9	30.8	0.4	0.0	0.0	0.0	0.0	85	85	35	0
$\Delta W(\Delta O)$	1.3	0.7	7.0	23.3	0.0	0.0	3.1	0.0	767	767	111	767
$\Delta W(\Delta P)$	1.5	22.5	10.2	102.9	0.0	0.0	0.0	4.1	767	767	112	767
ΔO	0.3	0.6	6.3	15.5	0.0	0.0	0.0	0.0	49	49	18	11
$\Delta O(\Delta W)$	0.4	0.7	116.3	31.7	0.0	0.0	0.2	3.5	85	85	49	26
ΔP	0.3	0.5	4.9	34.4	0.0	0.0	0.0	0.0	49	49	18	10
$\Delta P(\Delta W)$	0.5	16.0	45.9	518.2	0.0	0.0	0.0	4.7	85	85	49	26

Table 9: Computational results: Running Time, MIP Gap and Objective Value for all optimization criteria.

resolution time and the objective function value. Indeed, Table 9 shows that most of the solutions obtained are optimal and that the schedules with a MIP gap above 3.0% are schedules associated with a mobile fleet of shovels, which are more complex for the solver to obtain the schedule. We tried to solve all the schedules with a gap of 0, but it was not possible in some instances due to the high computational time.

6 Results and discussion

This section presents the results and discussion based on applying the optimization model proposed to the open-pit mine case study.

The resolution time, MIP gap, and objective function of the schedules are presented in Table 9. Overall, we observe that excepting the case $\Delta P(\Delta W)$, computation times are reasonable (less than two hours). We also observe an impact of $\times 2, \times 9$ or even unbounded on ΔW (deviation of removed waste material) if its optimization is secondary to production deviations ΔO or ΔP. In contrast, if the deviation in waste is first optimized and then ΔP or ΔO are optimized, the impact in production is significant. For example, the deviation in ΔP and ΔO augments from 49 to 85 (i.e. , a 73.5% increase) when the fleet is static. , and relatively more when the fleet is mobile, with an increase of 172% in ΔP and ΔO if there is no stockpile, and about 160% if there is a stockpile (again for both criteria). This implies that prioritizing waste deviation can have a huge impact in the value of the plans.

We compare the schedules obtained in terms of the compliance performance indicators presented in Table 2: waste movement (Γ_W), plant utilization (Γ_P), mine to plant (Γ_O), and mining extraction (Γ_M).

Table 10 presents the results in the case where the stocks are deactivated in the model. We observe that for the static fleet, when the priority is production (ΔP and ΔO), the hierarchical method cannot improve the single-criteria results, i.e., the performance of all strategies is equivalent, even in terms of waste extraction. However, if the fleet is allowed to move between sectors, then $\Delta W(\Delta O)$ and $\Delta W(\Delta P)$ generate plans which are equivalent for production and mining performance, but with a significant increase in waste extraction, improving the compliance from 65% to 94%. This improvement is important because the compliance of 65% means that the remaining 35% of waste would be left to be extracted in future periods, thus negatively impacting production for incoming months.

If the mine planner has the extraction of waste as a priority, then the hierarchical method outperforms ΔO. Indeed, the method improves compliance indicators associated to production (i.e., I_P, I_O) considerably. If the fleet is static, these indicators fall from 73% to 53%, instead of 20%. When the fleet is allowed to move between sectors, it can increase the production's compliance from 72% to 73%.

It is worth noting that having waste as the highest priority is not unrealistic. Whenever there is a shortage in equipment (for example, due to mechanical failures), mine operations prefer assigning equipment to ore mining faces; therefore, delaying waste removal. As this effect accumulates over time, planners eventually need to set waste extraction as a priority to not risk shortages of ore in the future. In such situations, the results suggests that using the hierarchical approach would help planners to get up to date in waste removal with a much better performance in terms of production, when compared to a single approach that only aims to maximize waste extraction.

Table 11 presents the results when stocks are enabled in the model. First, we observe that, as expected, stocks' availability positively impacts compliance overall. Second, the results are consistent with the ones for the case of Table 10:

– When waste extraction is a priority, the hierarchical method performs much better than ΔW (which sends no mineral to the plant).

Fleet	Static				Mobile			
Indicator	Γ_W	Γ_P	Γ_O	Γ_M	Γ_W	Γ_P	Γ_O	Γ_M
ΔW	96	20	20	20	98	72	72	72
$\Delta W(\Delta O)$	62	73	73	73	94	90	90	90
$\Delta W(\Delta P)$	62	73	73	73	94	90	90	90
ΔO	62	73	73	73	65	90	90	90
$\Delta O(\Delta W)$	96	53	53	53	98	73	73	73
ΔP	62	73	73	73	65	90	90	90
$\Delta P(\Delta W)$	96	53	53	53	98	73	73	73

Table 10: Performance of the different strategies in terms of compliance (%) , when no stocks are available.

– When production has the highest priority, ΔP reaches values of Γ_O and Γ_M, which suggests that it tends to rely on stocks more to achieve the same $\Gamma_P = 73\%$ than other strategies. Such a plan is risky because it may deplete the stockpile and also more expensive because of the extra cost of rehandling.

Overall, $\Delta W(\Delta O)$ and $\Delta W(\Delta P)$ are the strategies with the best performance in terms of production: for the static fleet, it produces the best results, and for the mobile fleet case, it reaches a value of $\Gamma_P = 94\%$, which is 1% lower than the maximum possible, but with the highest possible compliance of material being sent directly from the mine to the plant.

Fleet	Static				Mobile			
Indicator	Γ_W	Γ_P	Γ_O	Γ_M	Γ_W	Γ_P	Γ_O	Γ_M
ΔW	96	0	0	20	100	0	0	72
$\Delta W(\Delta O)$	62	73	73	80	62	94	94	95
$\Delta W(\Delta P)$	62	73	73	80	62	95	50	95
ΔO	62	73	73	73	62	94	94	95
$\Delta O(\Delta W)$	96	53	53	60	100	86	86	100
ΔP	62	73	44	73	62	95	68	95
$\Delta P(\Delta W)$	96	53	53	53	100	86	86	100

Table 11: Performance of the different strategies in terms of compliance, when stocks can be used.

Discussion

Overall, the model seems to abstract the mine operation in a proper manner and follow some expected behaviour. For example, in general terms, compliance increases with fleet mobility and with the possibility to use stockpiles. Also, it tends to favor compliance indicators for the corresponding optimization targets.

Aplying the hierarchical method to short-term planning generates plans that are more balanced and robust when compared to single objective optimization. This is because a single objective approach cannot take into account other criteria and, therefore, may produce plans that either have a poor impact during the planning horizon, or may hide problems for future periods of time.

The case study shows that the results can change substantially depending on the criteria used and their rank for the hierarchical method. We consider this, in fact, a feature of the mathematical model, which is flexible enough to incorporate all the criteria, therefore providing the planner with different choices, but also providing valuable information about potential issues. This is the case, for example, of the plan for ΔP and stockpiles (Table 11), which promises a high production compliance, but at the expense of delaying extraction from the mine.

The results are also interesting for the specific mine, because even in the best cases, there seems to be a trade-off between production and waste extraction. Indeed, the best strategies for production may reach over 90% compliance, but at the expense of having a 62% confidence for waste extraction, or conversely, when a $\Gamma_W = 100\%$ is reached, the compliance for production are $\Gamma_P = \Gamma_O = 86\%$).

Overall, the hierarchical method (especially $\Delta W(\Delta O)$ and $\Delta W(\Delta P)$) generates plans that are more robust, because they do not leave production or waste extraction for future periods and, therefore, promise a better adherence to long-term plans and goals.

7 Conclusions and future work

In this study, we propose a MILP optimization model to address the problem of shovel assignment and movement in an open-pit short-term mine context, where production and waste extraction targets have been set by long-term plans and stockpiles are available as material buffers. The model can be used with different optimization targets to implement single objectives or a hierarchical method that allows optimization of multiple criteria.

To evaluate the performance of the model with different criteria, we propose compliance performance indicators, which assess the quality of the plan generated in terms of how close planned tonnages are with regards to their targets. Thus, mine planners can use these indicators to evaluate and compare multiple short-term schedules.

We apply the proposed optimization model to a real-scale open-pit mine case study, over a time horizon of a month, in which mining faces are distributed over six mining sectors. The mine operation has one ore processing plant and uses six shovels and has one stockpile for operation. In this setting, we utilize the model to generate schedules under different scenarios; namely, single-optimization or hierarchical optimization of different short-term objectives, presence or absence of a stockpile, a mobile or a fixed shovel fleet. The objective is to study the impact of the different scenarios on the schedule indicators.

The results of the case study show that: (a) the hierarchical method can generate short-term mine production schedules optimizing the considered objectives, (b) when applying the hierarchical optimization method, both the objectives and the order of optimization of these have a great impact on the values of the different schedule indicators, (c) in general, schedules with a stockpile obtain higher schedule indicators compared to the ones with no stockpile, and

(d) schedules with a mobile shovel fleet obtain higher schedules' indicators than the ones with a fixed shovel fleet.

More importantly, in general, we observe that the plans generated using the hierarchical method are more robust, because they minimize potential delays on other criteria (not considered in the single objective approach). It is also interesting to see that by using all proposed strategies for optimization (single and hierarchical), the different plans can be analyzed in terms of their relative strengths and weaknesses. That is, the planner can choose from different plans, making a more informed decision.

As future work, we want to incorporate more aspects of the mining operation in the optimization model such as: scheduled shovel maintenance, allocation of drilling rigs to mining faces, multiple ore processing plants, multiple stockpiles, and grade blending in the stockpiles. We also intend to simulate the short-term mine production schedule generated by the optimization model. We plan to apply discrete-event simulation to assess the probability of compliance of the schedule. Finally, we plan to incorporate both geological uncertainty and equipment performance uncertainty in the optimization model.

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