Integrating construtability of a project into the optimization of production planning and scheduling

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Abstract

The complexity of the process of scheduling mine production in underground caving mines has forced the planners to split it into several disaggregated steps in which the economic envelope is determined without production capacity or time dynamics, the extraction method is chosen used highly aggregated data and raw calculations, the extraction sequence is aided by software, but defined by hand (looking for feasibility rather than maximum value), and the developments scheduling (what to build and when) is defined at the end.

The process described above not only leads to suboptimal plans (in terms of the overall value that is obtained from the mine resources), but it may actually produce infeasible production scheduling and therefore production goals that cannot be reached. One particular case in which this can be very important is that production scheduling and development scheduling are not integrated.

In this paper we introduce an optimization model that schedules both production and development at the same time, therefore producing combined production and development schedules which are optimized consistently over the considered time horizon. The model abstracts the construction and extraction in the mine as activities subject to: precedence constraints (which limit the relative order of activity completions), resource constraints (for example, on the available equipment to perform the activities), and aims to maximize the overall NPV of the project (that is the income from extraction activities minus the cost of development discounted over the time horizon). The model was implemented and used on real study cases that are presented in this article.

Introduction

Mine planning is the discipline of mining engineering that transforms the information on the resource, economic and market parameters and the owner directives into decisions that are summarized in the production plan, which establishes how much material is scheduled for extraction at each time period, and overall operation parameters like production goals, investment levels, and so on. Mine planning is also responsible for taking long or medium term decisions and transforms them into short term plans and schedules for a few days or weeks.

The above means that the overall mine planning process is complex, hence it is common practice to decompose it into different tasks so the overall process and specific plans for given decisions levels are constructed independently and more easily. Unfortunately, this disaggregation of the mine planning process into different steps means that the final schedules do not necessarily capture the real value of a project. Indeed, as the steps in the mine planning process are carried sequentially, former decisions are made with highly aggregated information and models that do not capture the complexity of forthcoming steps, and later decisions are subject to the initial one, hence the overall result is suboptimal.

The particular example that motivates this paper comes from the decoupling of determining the extraction scheduling (that is, what is to be mined from a underground mine and when), from the development scheduling (which is the set of construction activities and infrastructure to be carried on in order to realize the extraction scheduling and corresponding production goals. Indeed, examples can be found of mines that cannot fulfill their production's budgets due to the inability to fulfill infrastructure development requirements. For instance, Díaz and Morales 2008 indicates that in 2002 they had a 61% fulfillment of development and a 70% fulfillment of production, highlighting the importance of the topic.

Main modeling aspects

In this section we briefly present the main elements considered by the model.

Maximum development rate: This states the maximum feasible quantity of metres that can be performed at any given time for any activity (production unit, section of a tunnel, etc). In this model we consider this maximum development as the one given by nature, that is, if the construction and production resources are unlimited; and transform it into a maximum percentage of the activity to be performed.

Cost or profit: These take place in the goal function to be maximized. Positive values (profits) are associated to production activities (but can be negative values (costs) to development activities (notice that, depending on the ore content, there could be production activities with a net value that is negative).

Resources: These are essential for the correct or real analysis because they indicate which materials, machines, workers or time are necessary to complete an activity. For example, a given section of a tunnel may require a certain number of jumbo machine hours, another amount of Tunnel Boring Machine hours, and similarly borer machine time. Conversely, there exists (at each moment) an overall availability of these resources, that must be shared between the activities that require it.

Physical and Operational precedences: These relations define what developments must be constructed in order to gain access (physical) or allow starting other activities. These constraints depend on the layout of the mine, which is assumed to be fixed (Rahal & Smith, 2003, Newman & Kuchta, 2007).

Production constraints: These state the parameters that have to obey certain mining methods. In block caving there is a draw rate, which controls flow of muck, and the draw ratio already mentioned. This will control the dilution entry point and damage to the production level. Most importantly, it gives a space consistency in relation to the production activities (Rubio & Diering, 2004).

This paper presents an optimization model (a mixed integer problem, MIP, to be precise) that allows schedule of both: the extraction and the development, of an underground mine, so that the value of the project is maximized under resource and precedence constraints over the different development and extraction activities. The model is an extension on the model described in Rocher et al (2011) and has been implemented into a software prototype which also contains specialized algorithms to improve the execution time of the computation of an optimal schedule (in terms of the overall NPV). The model considers a number of activities, each with the following attributes (others can be specified):

- Minimum and maximum rates (in fraction per time).
- Overall net value (which is negative for construction activities, and can be positive or negative for extraction activities, depending on the ore content).
- Starting and Ending costs (that do not depend on the progress of a given activity, but on the decision of starting them at a certain time period, and respectively ending them).
- Resource consumption (for any given resource, like equipment availability, a certain amount is consumed for each fraction of progress in the activity).
- Precedences: for each activity A, there exists a set of activities that need to be finished before A starts.

Given these activities and their attributes, as well as the overall set of periods (which can be of different lengths), and the overall resource availability (per resource and period), the models aims to maximize the NPV of the project, that is, the discounted values coming from revenue of extraction activities minus the costs of development.

Using a mixed integer programs for sequencing underground mines is not new. For example, Newman et al (2007) and Martinez & Newman (2011) present models and algorithms for scheduling sublevel stoping activities, O'Sullivan (2010) presents a model and algorithms for a complex underground mine operation involving several explotation methods, and Morales et al (2009) presents a model for sublevel caving. Nevertheless, while these models consider precedence and construction limitations, they are extraction-oriented. A longer review on operation research optimization models tackling scheduling of underground mines can be found in Newman et al (2010).

The model described in this article is more general than the ones described above, as it abstracts the mine in a set of activities, hence it does not have special considerations regarding the extraction method. Similarly, the model could be adjusted to different time-scales, having for example production activities concerning a single stope or full block in a block caving mine.

Conversely, the Scheduling Optimization Tool (SOT) developed by MIRARCO (Maybee 2008) optimizes scheduling extraction and development. In this case the optimization technique is based on genetic algorithms. Unfortunately, while the results of using this technique are promising in terms of speed and quality of the solutions (they improve on human constructed ones) they do not guarantee optimality of the final schedule; this is not the case of binary linear programs, in which it is possible to construct upper bound on the optimal NPV and therefore have an estimation of the quality of the final solution.

Mathematical modeling

In this section, we introduce the mixed integer program for the activity scheduling problem.

We consider time discretized into $\boldsymbol{t} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{T}$ time periods or time slots, where $\boldsymbol{T}$is the *time horizon* or the number of periods for scheduling. Without loss of generality, we assume that each time period lasts 1 unit of time.

Activities and economic parameters

A set $\boldsymbol{A}$ of activities must be considered in order to fulfill the entire mine design. Then, for activity $\boldsymbol{i} \in \boldsymbol{A}$, we denote as $\boldsymbol{v}_{\boldsymbol{i}}^{+}$ the cost for starting the activity, $\boldsymbol{v}_{\boldsymbol{i}}^{-}$ cost of finishing/closing and $\boldsymbol{v}_{\boldsymbol{i}}$ the net profit/cost of developing.

We also consider $v_{\max i}$ the maximum rate of progress for activity i, and conversely $\ell_i = \frac{1}{v_{\max i}}$ the length of the activity (recall that we assumed that each time period lasts one unit of time).

Decision variables

The decision variables are:

$$\boldsymbol{p}_{it} = \ \mathit{percentage\ of\ activity}\ \boldsymbol{i}\ \mathit{developed\ at\ time\ period}\ \boldsymbol{t}.$$

The following are associated with the beginning and, similarly, with the ending of an activity:

$$\boldsymbol{s}_{it} = \begin{cases} 1 & \text{activity } \boldsymbol{i} \text{ has started by time-period } \boldsymbol{t}, \\ 0 & \text{if not.} \end{cases}$$

$$\boldsymbol{e}_{it} = \begin{cases} 1 & \text{activity } \boldsymbol{i} \text{ has not yet ended by time-period } \boldsymbol{t}, \\ 0 & \mathit{if\ not}. \end{cases}$$

Objective function

Now that the basic ideas have been defined, the objective function that maximizes the overall net profit, discounted by a factor $\boldsymbol{\alpha} < 1$ that shows the time effect depending on the assumed risk, can be formulated as follows:

$$\boldsymbol{V} = \sum_{\boldsymbol{t}=1}^{\boldsymbol{T}} \boldsymbol{\alpha}^{\boldsymbol{t}} \sum_{\boldsymbol{i} \in \boldsymbol{A}} (\boldsymbol{v}_{\boldsymbol{i}} \boldsymbol{p}_{\boldsymbol{it}} - \boldsymbol{v}_{\boldsymbol{i}}^{+} \boldsymbol{\Delta s}_{\boldsymbol{it}} - \boldsymbol{v}_{\boldsymbol{i}}^{-} \boldsymbol{\Delta e}_{\boldsymbol{it}}) \tag{1}$$

Where $\Delta s_{it} = s_{it} - s_{it-1}$, $\Delta e_{it} = e_{it} - e_{it-1}$ and we set $s_{i0} = e_{i0} = 0$.

Constraints defining variables structural relations

There are some basic definitions regarding the decision variables that make sure that they are well represented clearly. In this case, the Equation (2) shows that there is only one start time and one end time. Equation (3) and (4) means that for develop one segment, it has to be started and not yet finished, meanwhile, at the same time, the progress at any given period cannot be greater than 100% of the total progress.

$$\boldsymbol{\Delta s}_{it} \geq 0, \qquad \boldsymbol{\Delta e}_{it} \leq 0 \qquad (\forall \boldsymbol{i} \in \boldsymbol{A})(\forall \boldsymbol{t} = 1,2, \dots, \boldsymbol{T}) \tag{2}$$

$$\boldsymbol{p}_{it} \leq \boldsymbol{s}_{it} \qquad (\forall \boldsymbol{i} \in \boldsymbol{A})(\forall \boldsymbol{t} = 1,2, \dots, \boldsymbol{T}) \tag{3}$$

$$\boldsymbol{p}_{it} \leq \boldsymbol{e}_{it} \qquad (\forall \boldsymbol{i} \in \boldsymbol{A})(\forall \boldsymbol{t} = 1,2, \dots, \boldsymbol{T}) \tag{4}$$

$$1 - \boldsymbol{e_{it}} \leq \sum_{s \leq t} \boldsymbol{p_{is}} \qquad (\forall \boldsymbol{i} \in \boldsymbol{A})(\forall \boldsymbol{t} = 1,2, \dots, \boldsymbol{T}) \tag{5}$$

$$\boldsymbol{p_{it}} \leq \boldsymbol{v_{max\,i}} \qquad (\forall \boldsymbol{i} \in \boldsymbol{A})(\forall \boldsymbol{t} = 1,2, \dots, \boldsymbol{T}) \tag{6}$$

Additionally, Equation (5) says that to end an activity it is necessary to do 100% of the activity, and finally Equation (6) sets the maximum development rate for all segments like was already defined.

Constraints on the resources

We take a set $\boldsymbol{R}$ of available resources and a required resource $\boldsymbol{c_r^i}$ of resource $\boldsymbol{r} \in \boldsymbol{R}$ for completing activity $\boldsymbol{i}$. The overall availability of resource $\boldsymbol{r}$ at time-period $\boldsymbol{t}$ is denoted as $\boldsymbol{R_t^r}$. The constraints simply establishes that the usage of a resource $\boldsymbol{r}$ overall activities that progress at a given time period and use such resource, cannot exceed the total availability of the resource at the given time period.

$$\sum_{i \in A} \boldsymbol{c_r^i p_{it}} \leq \boldsymbol{R_t^r} \qquad (\forall \boldsymbol{i} \in \boldsymbol{A})(\forall \boldsymbol{r} \in \boldsymbol{R})(\forall \boldsymbol{t} = 1,2, \dots, \boldsymbol{T}) \tag{10}$$

Constraints representing precedences

For each activity $\boldsymbol{i}$, we consider a set $\boldsymbol{P(i)}$ of predecessors, that is $\boldsymbol{j} \in \boldsymbol{P(i)}$ means that activity $\boldsymbol{j}$ must be finished before (or at the same time period) and $\boldsymbol{i}$. Notice that this kind of relation induces a directed graph $G = (\boldsymbol{A}, \boldsymbol{X})$ where $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{X} \Leftrightarrow \boldsymbol{j} \in P(\boldsymbol{i})$, hence we can talk about *root nodes*: $\boldsymbol{i} \in A$ is a root if $P(\boldsymbol{i}) = \emptyset$ (i has no predecessors). We call $\boldsymbol{\Lambda}$ the set of all roots; and *leaf nodes:* $\boldsymbol{i} \in \boldsymbol{A}$ is a leaf if $\boldsymbol{i} \notin \boldsymbol{P(j)}$ for any $\boldsymbol{j} \in \boldsymbol{A}$ (i is not a precedessor). Let us call $\boldsymbol{\Gamma}$ the set of leaves.

We impose the precedence relation using two set of constraints. The first set of constraints simply establishes that in order to start a certain activity, all predecessors must be finished:

$$\boldsymbol{s_{it}} \leq \boldsymbol{1} - \boldsymbol{e_{jt}} \qquad (\forall (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{X})(\forall \boldsymbol{t} = \boldsymbol{1}, \dots, \boldsymbol{T}) \tag{11}$$

This set of constraints is not enough to capture the precedence because only the time required by the direct predecessor of an activity is considered. In order to solve this issue, we calculate, for each leaf $\boldsymbol{i} \in \boldsymbol{\Gamma}$ the set of all paths $\boldsymbol{\phi(i)}$ in G that start in i and end at some $\boldsymbol{j} \in \boldsymbol{\Lambda}$. Then we impose the constraint

$$\sum_{j \in \boldsymbol{Q}} \boldsymbol{\ell_j p_{jt}} \leq 1 \qquad (\forall \boldsymbol{i} \in \boldsymbol{\Gamma})(\forall \boldsymbol{Q} \in \phi(\boldsymbol{i}))(\forall \boldsymbol{t} = \boldsymbol{1}, \dots, \boldsymbol{T}) \tag{12}$$

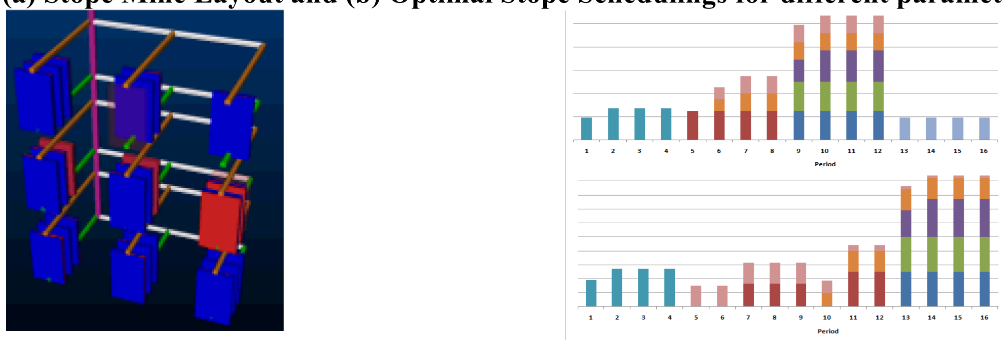
Numerical Experiences

In this section we present some applications of the model implementation. The first example is from a synthetic stope mine, which serves as illustration of the model results and has partially appeared in Rocher et al (2011). The second example is a real mine layout.

Example 1 – Stope Mine

This mine consists of 27 stopes, distributed in 3 levels of 9 stops each, giving an overall of 106 activities to be performed in 16 time periods. Development of shafts, tunnels and crosscuts is considered to gain access to each of the different stopes. The data (economic value) of each stope as well as costs and resource consumption for the stopes and development structures has been generated artificially, but within ranges expected from the literature and experience.

Figure 1. (a) Stope Mine Layout and (b) Optimal Stope Schedulings for different parameters



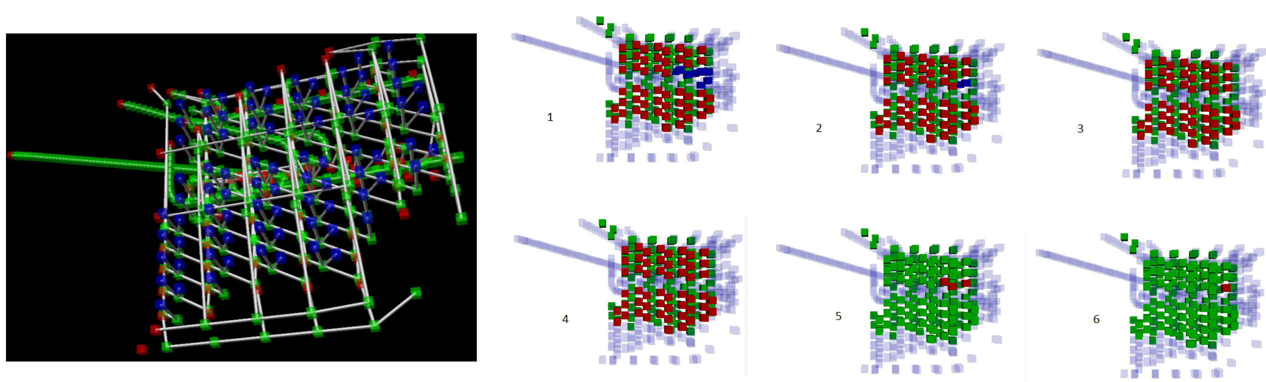
(a) (b)

Figure 1 (a) shows the layout of this small example, while Figure 1(b) shows the composition of the optimal production of the mine (thus, the extraction scheduling) colored by stope origin for two different settings of the drifting speeds. We note the optimal extraction sequence changes with the parameters.

Example 2 – Panel Caving Mine

This example comes from a small sector of a real mine. The instance is constructed using 2 sources of information: (a) economic envelope and economic heights for each column of material were calculated using PCBC, which also reports the tonnage involved for each column of material (we ignore the scheduling generated by PCBC). There are 85 columns to be scheduled for extraction up to 14 years, and (b) development structures and their precedences that were modeled using Mine 2-4D, from which a text file describing these structures, maximum rates and other parameters. There are 574 development activities. The two components are then linked associating, to each column, the closest launder in the development structures.

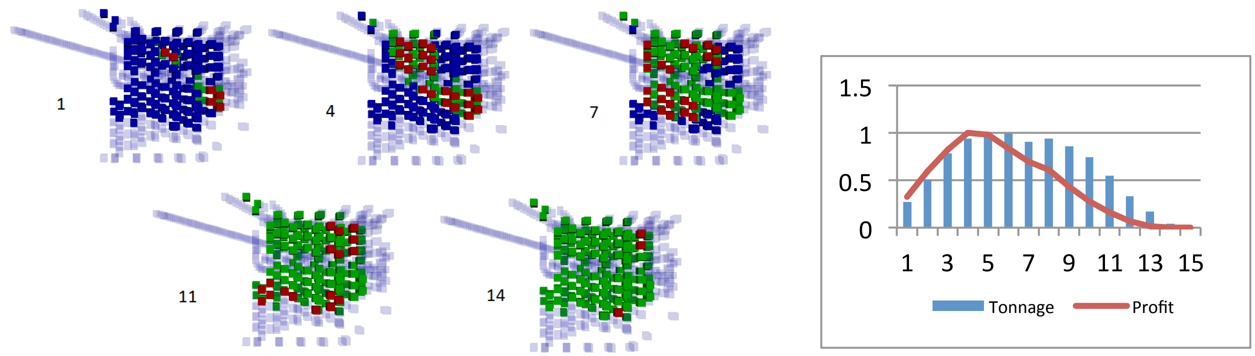
Figure 2. Schematic mine layout and a production and extraction scheduling.



(a) (b)

Figure 2 (a) a schematic view of the mine sector in which activities are represented by boxes and precedences links between them. Red boxes are activities without predecessors, and blue boxes represent production activities. Figure 2 (b) presents the scheduling resulting from a model run (which takes a few minutes on a standard notebook)- Translucent activities are not developed, blue activities have not been started, red activities are in progress and green activities are finished. Notice this is a run without resource constraints on the development activities, nor in the number of active drawpoints (there is only an overall capacity in tonnage), thus the optimizer constructs the minimum to access profitable columns and can do it very quickly, ending after 6 annual periods.

Figure 3. An resumed scheduling and associated production plan



(a) (b)

Figure 3(a) shows the results in terms of scheduling for a different run, in which we imposed constraints on the availability of construction resources as well as overall number of active drawpoints, for the periods specified by the small numbers. The colors have the same meaning as in Figure 2. Figure 3(b) presents the corresponding production plan, normalized on the capacity (blue bars, with the overall tonnage) and for and cash flows (normalized on the maximum period).

Conclusions

We have presented an extended version of a scheduling model for production and development of underground mines. The model abstracts the mine a set of activities which have to be scheduled under resource and precedence constraints, which makes it general and versatile, allowing different applications. A first result of the runs shows that the optimal production scheduling changes when parameters and construction constraints changes, which is very important to consider as the current practice is to optimize the production scheduling and fix it before the developing schedule is constructed.

The model is also very promising in terms of speed, therefore opening possibilities of use in very large mine systems as well as considering different scenarios of uncertainty, for which many runs of the model must be done.

While not presented in this article, we have already worked on improvements on the model. For example, the path constraints required for expressing the precedences can be replaced using additional variables. This is very important, as using the path constraints can become unfeasible for large mines. Finally, we are currently working on a deeper and detailed industrial validation of the results, which we expect to achieve in the near future.

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