A comparison between conventional nested pits based on LG and an extended approach for generating nested pits in open-pit mine production

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Abstract

Long-term open-pit production planning is traditionally carried out using the nested pit methodology, the fundamentals of which were established in 1965 by Lerchs and Grossmann (LG). A phase design is generated from the set of nested pits, which is essential for scheduling production. However, this phase design has limitations: (i) in what is commonly known as the gap problem, there may be large size differences between consecutive phases; and (ii) time is not considered in its definition, heavily conditioning aspects that include the next stage of scheduling the production of blocks. In this article, we present an extension of the traditional nested pits based on the LG methodology in order to generate phase selection based on two ideas: (i) limiting the depth of the pits obtained; and (ii) including the temporal dimension in its definition. One advantage of this proposal is that it can improve the phase selection process, increasing the options for open-pit mine design, focusing the mine planner's efforts on assessing various scenarios and reducing the time employed in the planning task.

1 Introduction

Mining planning is one of the fundamental disciplines of the mining business. A mining plan determines the return on a project by defining how and when to extract mining reserves to quantify the life of the mine, the optimal production rate and the human, economic and technical capital necessary. This, in turn, generates the company's business plan, which manifests itself as the production schedule.

The information must be analysed at different stages to arrive at the production schedule, defining how the mining reserves will be exploited and ensuring an operational design that is in accordance with reality and maximises the project's net present value (NPV).

Figure 1 shows the different stages of the process, from orebody location to the production schedule, according to the traditional methodology (Morales et al. 2019).

Currently, there are two methodologies for long-term open-pit production planning:

- Traditional method (LG): This approach is based on the Lerchs-Grossmann algorithm (Lerchs and Grossmann 1965) and the generation of nested pits.
- Direct block scheduling method (DBS): This approach to generating production schedules seeks to schedule blocks directly through mathematical programming models, taking time and operational aspects into account as constraints.

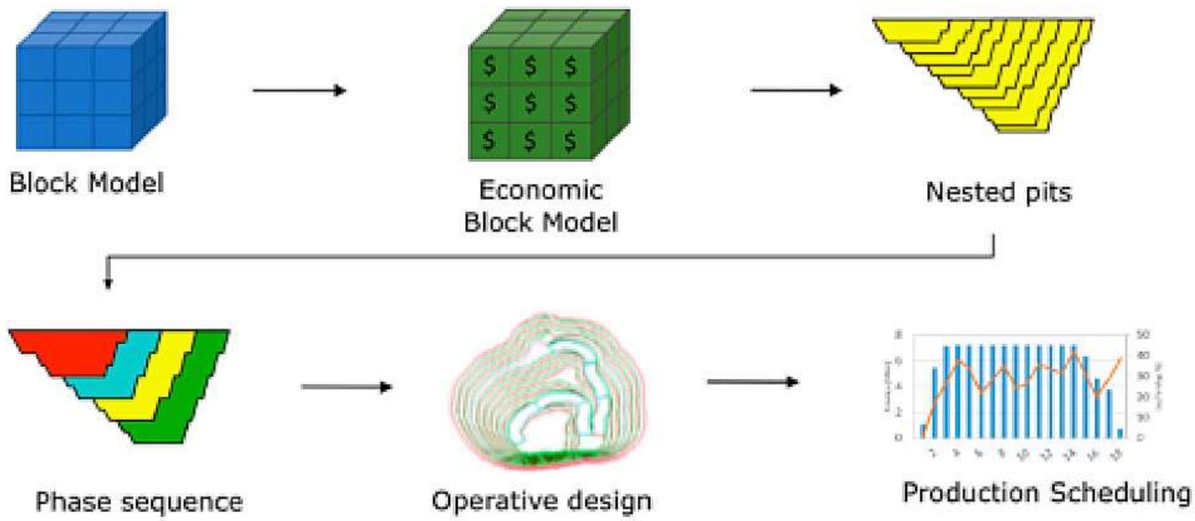

Figure 1 **Stages in the long-term planning of a deposit from the block model to the production schedule according to the traditional methodology (Morales et al. 2019)**

Figure 2 shows a summary of the two approaches and their advantages and disadvantages are summarised in Table 1.

A valuation of the block model is required to obtain the economic model by taking into account different parameters such as metal price, mine, processing and selling costs, metallurgical recovery and ore grades.

By using a marginal cut-off grade, Equation (1) describes the calculation of profit in block b:

$$v_b = \begin{cases} \left[(P - C_v) \cdot \text{Rec} \cdot \text{f} \cdot y_b - C_m - C_p\right] \cdot ton_b & , if\ y_b \geq \dfrac{C_p}{(P - C_v) \cdot \text{Rec} \cdot \text{f}} \\ -C_m \cdot ton_b & , otherwise \end{cases} \quad (1)$$

Where:

P = metal price

C_m = mine cost

C_p = plant cost

C_v = selling cost

Rec = metallurgical recovery

F = a suitable unit conversion factor

y_b = ore grade

ton_b = tonnage for block b.

Under the traditional methodology, once the block model has been valued, the final pit is generated based on the LG algorithm (Lerchs & Grossmann 1965), respecting geometric constraints given by the slopes of the pit walls (Espinoza et al. 2013). Once the final pit is obtained, the nested pits are generated within it. They correspond to a pit family that is smaller than the final pit, parameterizing the metal price P in Equation (1) by a factor known as revenue factor or λ.

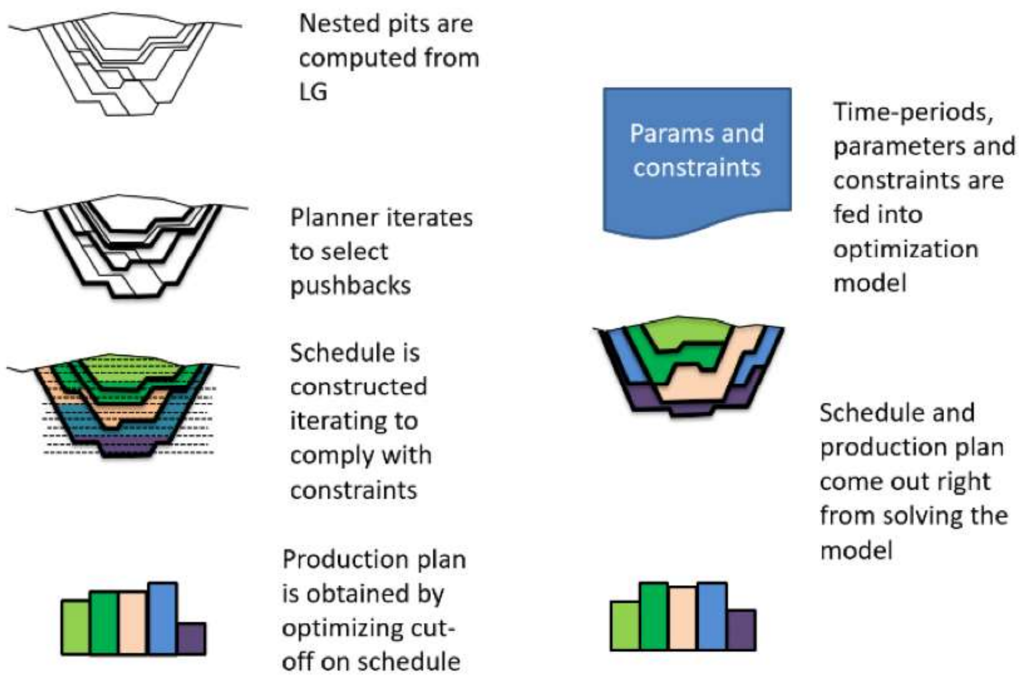

Figure 2 **Stages of the traditional methodology based on LG (left) and DBS (right) (Morales et al. 2015)**

Table 1 **Advantages and disadvantages of LG and DBS approaches**

Method	Advantages	Disadvantages
LG	- Widely used in industry - Operability of results	- Manual phase selection criterion and gap problem - Iterative process
DBS	- Generates schedules directly - Considers time and opportunity cost	- New approach, slow assimilation by industry - Considerable computation time in real case studies

Equation (2) incorporates the revenue factor λ in block valuation (Jélvez et al. 2020).

$$v_b^\lambda = \begin{cases} \left[(P \cdot \boldsymbol{\lambda} - C_v) \cdot \text{ Rec} \cdot \text{ f} \cdot y_b - C_m - C_p\right] \cdot ton_b & , if\ y_b \geq \dfrac{C_p}{(P - C_v) \cdot \text{Rec} \cdot \text{f}} \\ -C_m \cdot ton_b & , otherwise \end{cases} \quad (2)$$

Once the nested pits have been obtained, they must be grouped according to specific criteria that are usually extremely dependent on the mining engineer's experience. This selection is known as phase or mine expansion (Figure 3). These phases determine how ore extraction will be developed. From this selection, the scheduling and the final profit of the project are determined using a bench-phase extraction criterion.

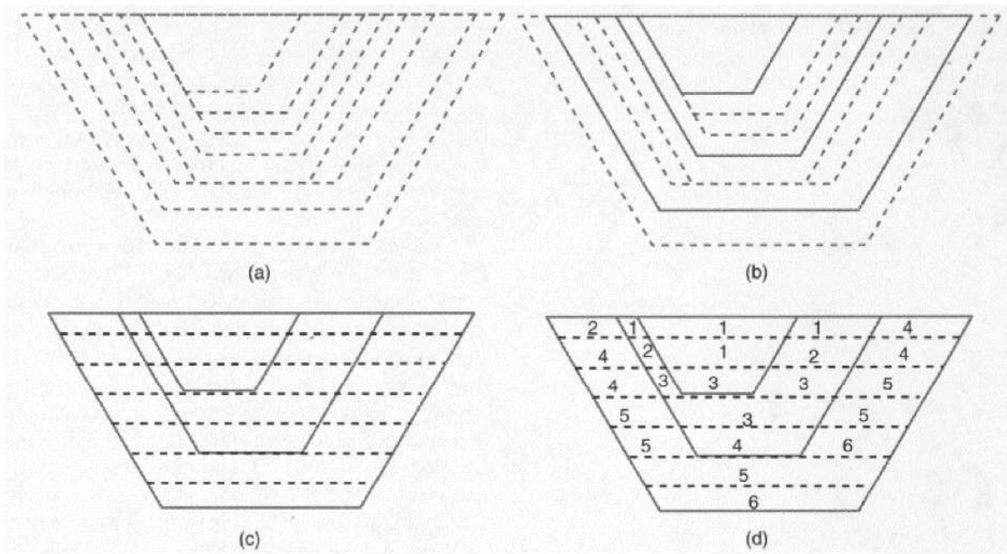

Figure 3 Sequence for generating the production schedule from final pit: (a) a sequence of nested pits is defined; (b) phases are selected from the nested pits generated; (c) bench-phases are defined; (d) bench-phases are scheduled for production in several time-periods (Chicoisne et al. 2012)

Despite the research into new algorithms such as pseudoflow (Hochbaum 2008) that has been undertaken in a bid to reduce computation time in pit generation, problems with this approach persist: (i) the time value of money is not considered directly; and (ii) it is not possible to control the size of pits obtained, limiting the elaboration of a good selection of phases (gap problem). Figure 4 shows the gap problem in the sense that there is no revenue factor for identifying intermediate pits between P_2 and P_3.

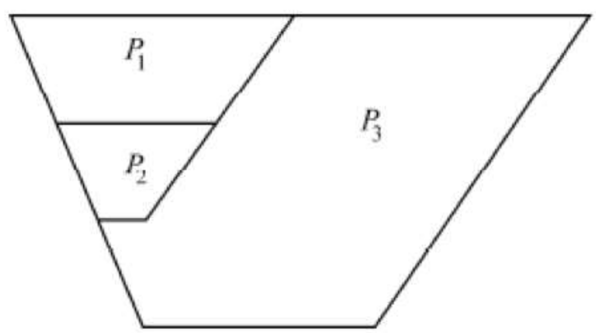

Figure 4 Representation of the gap problem (Meaguer et al. 2014)

The gap problem has been studied in a bid to find a solution that reduces the gaps and offers better control over phase design in order to obtain the highest possible operative NPV in the production scheduling stage (Meaguer et al. 2014).

One possible solution to the gap problem may be to limit the pits' depth, making them closer to each other and avoiding considerable spacing, which would improve phase selection.

As indicated above, another problem is the opportunity cost that can arise from exploiting a block without considering the time dimension due to the NPV. The actual value of block b depends on the period t in which it is extracted, considering discount rate i:

$$v_{bt} = \frac{v_b}{(1+i)^t}$$

In other words, the profit from extracting an ore block decreases with the postponement of exploitation. This is not taken into account under the traditional methodology so blocks that are perhaps further from the surface are often ignored but may represent higher profits with respect to extraction time (Chicoisne et al. 2012).

The next section sets out two alternatives to the traditional LG algorithm that could be used to control the size of intermediate pits and, thus, reduce the impact of the gap problem in phase selection.

2 Methodology

The main steps of the methodology can be summarised as follows:

i. Final pit generation: For a given block model, an economic model as given by Equation (1) and a pit slope design, the final pit is generated by using a customised version of the pseudoflow algorithm (Section 2.1).

ii. Intermediate pit generation through:

   a. revenue factors (Section 2.2.1). This is considered as a base case.

   b. expected extraction time (Section 2.2.2).

These approaches are combined with another strategy: that of limiting the depth of pits. All these new pits are input into the phase selection process.

2.1 Final pit generation

The following model is considered for computing a final pit:

$$(UPIT) \quad Max \ \sum_{b \in B} v_b \cdot x_b \tag{3}$$

subject to:

$$x_b \leq xb \qquad \forall \ b \in \mathrm{B}, \mathrm{b'} \in B_b \tag{4}$$

$$x_b \in \{0,1\} \qquad \forall \ b \in \mathrm{B} \tag{5}$$

where:

B = set of blocks, i.e. the block model

b = a block in the block model ($b \in B$)

B_b = set of blocks that are predecessor blocks of block . Equation (4) models precedence constraints in the sense that, in order to extract block b, block $b' \in B_b$ must first be extracted.

v_b = economic value of block b, as presented in Equation (1). Equation (3) shows the total value from the final pit.

x_b = binary variable: equal to 1 if block b is considered within the final pit; 0 otherwise (Equation (5))

($UPIT$) is solved by using the LG or pseudoflow algorithm.

2.2 Intermediate pit generation

In this section, we describe some strategies for computing intermediate pits. These new pits, smaller than the final pit, may be used to select those that, based on technical criteria, minimise differences in tonnage between pits (reducing the gap), are more consistent to group together and, thus, generate the phases for production scheduling.

2.2.1 Intermediate pits through revenue factors (RF) and limited depth

For a given revenue factor λ ($0 < \lambda \leq 1$) and a maximum permitted depth z^*, new intermediate pits can be generated.

$$(UPIT_{z^*}^{\lambda}) \quad Max \sum_{b \in B(z^*)} v_b^{\lambda} \cdot x_b \tag{6}$$

subject to:

$$x_b \leq x_{b'} \qquad \forall\, b \in B\,(z^*),\, b' \in B_b \tag{7}$$

$$x_b \in \{0,1\} \qquad \forall\, b \in B(z^*) \tag{8}$$

where:

$B\,(z^*)$ = subset of blocks such that the -coordinate is greater than the threshold z^*; note that $(z^*) \subseteq B$

v_b^{λ} = economic value of block b when the metal price is scaled by the revenue factor λ as presented in Equation (2).

The number of intermediate pits generated with this methodology is, at most, equal to the number of revenue factors times the number of benches of the block model.

Here, we seek to limit the depth of the final pit, truncating the original model from the upper levels, in order to obtain new models at different depths. For each of these models, the corresponding pits are determined, thus obtaining new intermediate pits with different depths and tonnages. It is worth noting that the technique of generating an optimal pit, limiting block model by depth, was implemented in the Whittle commercial software (Geovia 2020) for use as an alternative when 'revenue factors mode' is not available.

When $z^* = z^{min}$, where z^{min} is the deepest bench, $B(z^* = z^{min}) = B$, giving the same results as the traditional methodology based on the LG algorithm. Henceforward, we refer to this result as the base case.

2.2.2 Intermediate pits through expected extraction periods and limited depth

In this methodology, the main approach considers information regarding time. In other words, the discount rate and the economic impact on the final benefit of each block are taken into account, thus maximising the NPV. For this, the fixed cut-off grade version of the relaxed open-pit mine production scheduling problem ($R\text{-}OPBSP$) was considered, but limiting the maximum depth in the scheduling:

$$(R\text{-}OPBSP) \;\; Max \sum_{b \in B(z^*)} \sum_{t \in T} v_{bt} \cdot (x_{bt} - x_{bt-1}) \tag{9}$$

subject to:

$$x_{bt} \leq x_{b't} \qquad \forall\, b \in B(z^*),\, b' \in B_b,\, t \in T \tag{10}$$

$$x_{bt} \leq x_{bt-1} \qquad \forall\, b \in B(z^*) \tag{11}$$

$$C_{rt}^{-} \leq \sum_{b \in B(z^*)} q_{br} \cdot (x_{bt} - x_{bt-1}) \leq C_{rt}^{+} \qquad \forall\, t \in T, r \in R \tag{12}$$

$$x_{bt} \in [0,1] \qquad \forall\, b \in B(z^*),\, t \in T \tag{13}$$

where:

T = horizon planning

t = period ()

R = set of resources (for example, mining or processing)

C_{rt}^{-}, C_{rt}^{+} = minimum, maximum resource consumption, per resource and period

qbr = attribute to be limited, per block and resource (for example, mining tonnes, processing tonnes)

x_{bt} = continuous variable, the portion of block extracted by period .

Equation (9) maximises the NPV. Equation (10) corresponds to the precedence constraints. Equation (11) corresponds to each block which is extracted once. Equation (12) corresponds to the minimum/maximum capacity constraint on resources consumption R. Finally, Equation (13) corresponds to the nature of the variables, that is, a fraction of the block can be extracted.

The BZ algorithm (Bienstock & Zuckerberg 2010) was used to solve (R-$OPBSP$), which corresponds to a relaxed form of the usual $OPBSP$ (Jélvez et al. 2016). In other words, it does not assign integer periods to each block but determines approximate extraction periods and is used as a guide to the generation of intermediate pits.

If x^{*}_{bt} is the optimal solution of (R-$OPBSP$), then the expected extraction period $ET_b^{z^*}$ for each block $b \in B(z^*), \forall z^* \in \{z^{min}, \ldots, z^{max}\}$ is given by:

$$ET_b^{z^*} = \sum_{t=1}^{T} t \cdot (x^*_{bt} - x^*_{b,t-1}) + (T+1) \cdot (1 - x^*_{bt}) \qquad \forall\, b \in B(z^*) \tag{14}$$

From expected extraction periods a set of pits , is generated as follows (Jélvez & Morales 2017):

$$P_k^{z^*} = \left\{ b \in B(z^*) : \frac{(k-1)\cdot T}{n_0} < ET_b^{z^*} < \frac{k \cdot T}{n_0} \right\} \tag{15}$$

By applying this procedure, the number of intermediate pits generated with this methodology is, at most, equal to n_o times the number of benches of the block model.

3 Data

Arizona's copper deposit (KD) was used as a case study for application of the proposed methodology (Figure 5). The data was obtained from Minelib (Espinoza et al. 2013), a public library of databases of open-pit mines, available at http://mansci-web.uai.cl/minelib/kd.xhtml. The block model has several attributes, such as rock tonnage, ore tonnage and copper grade.

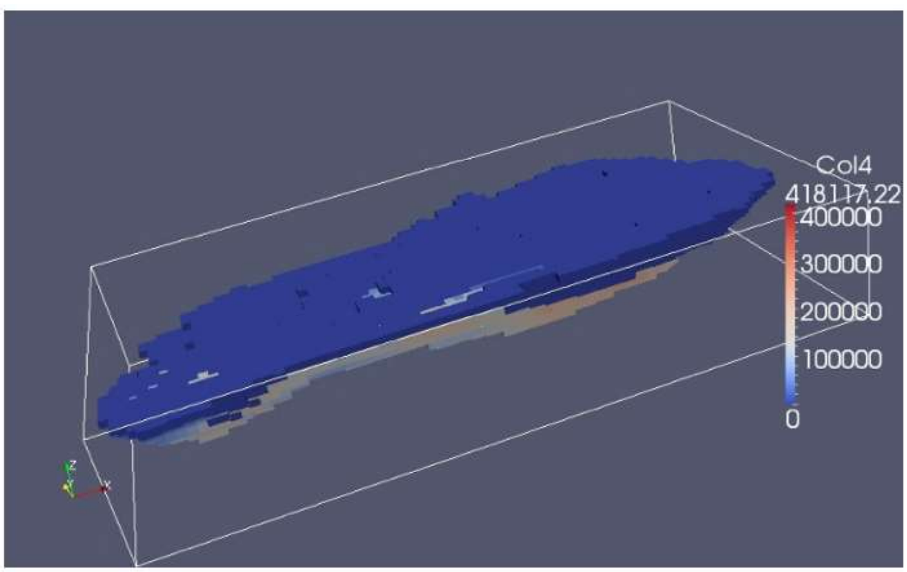

Figure 5 Isometric view of KD deposit (Minelib)

In all cases, geotechnical constraints (slope pit walls) are given by an overall angle of 45° and eight benches above (precedence cone).

To generate an economic block model

Table 2 shows the technical and economic parameters for economic valuation of the KD block model.

Table 2 Technical and economic parameters for economic valuation

Symbol and unit	Value	Description
P [USD/lb]	2.7	Metal price
C_v [USD/lb]	0.5	Selling cost
C_m [USD/ton]	4.0	Mining cost
C_p [USD/lb]	9.0	Processing cost
R	0.90	Metallurgical recovery

To delimit the depth of the block model

The KD deposit has 19 benches, $z^* \in \{500,515,530,\ldots,770\}$, with a bench height of 15 m. The depth is limited by considering one bench $B(z^* = 770)$ of the block model. In the next iteration, two benches $B(z^* = 755)$ of the block model are considered, and so on, until considering the entire block model $B(z^* = 500) = B$.

To generate nested pits by using revenue factors

Nested pits are calculated by considering a family of 100 revenue factors ($\lambda j = j/100$, for $j = 1, \ldots, 100$).

To generate intermediate pits by using expected extraction periods

Table 3 shows the parameters for solving (*R-OPBSP*) and expected extraction periods.

Table 3 Parameters to compute expected extraction periods

Symbol and unit	Value	Description
T [year]	12	Horizon planning, number of periods
i	0.1	Discount rate
$C^{-}_{min,t}/C^{+}_{min,t}$ [Mt]	0.0/20.0	Min/max mining capacity
$C^{-}_{proc,t}/C^{+}_{proc,t}$ [Mt]	0.0/10.0	Min/max processing capacity
n_o	12	Number of intermediate pits

The algorithms were implemented using the MineLink library (Minelink 2013) to generate final and intermediate pits through application of the pseudoflow algorithm (Section 2.2.1) and a customised version of the BZ algorithm (Section 2.2.2). DOPPLER software was used to visualise the results. The experiments were performed on an average-user desktop computer.

4 Results

In this section, the results are shown in terms of the new intermediate pits generated by combining (i) the Lerchs-Grossmann methodology and limited depth; and (ii) cumulative expected extraction periods and limited depth.

4.1 Extended intermediate pits through revenue factors limiting depth

By applying the model of Section 2.2.1 successively, a family of intermediate pits was generated for a set of RFs and depths. Figure 6 shows several views of the intermediate pits obtained by RF when depth is limited at different values ($z^* = 635m, 560m, 500m$). Clearly, for a given depth, the set of pits generated through RF are nested. Similarly, when setting an RF value, the set of pits generated while modifying the depth is also nested. An additional advantage when the depth is limited is the operational space at the pit bottom.

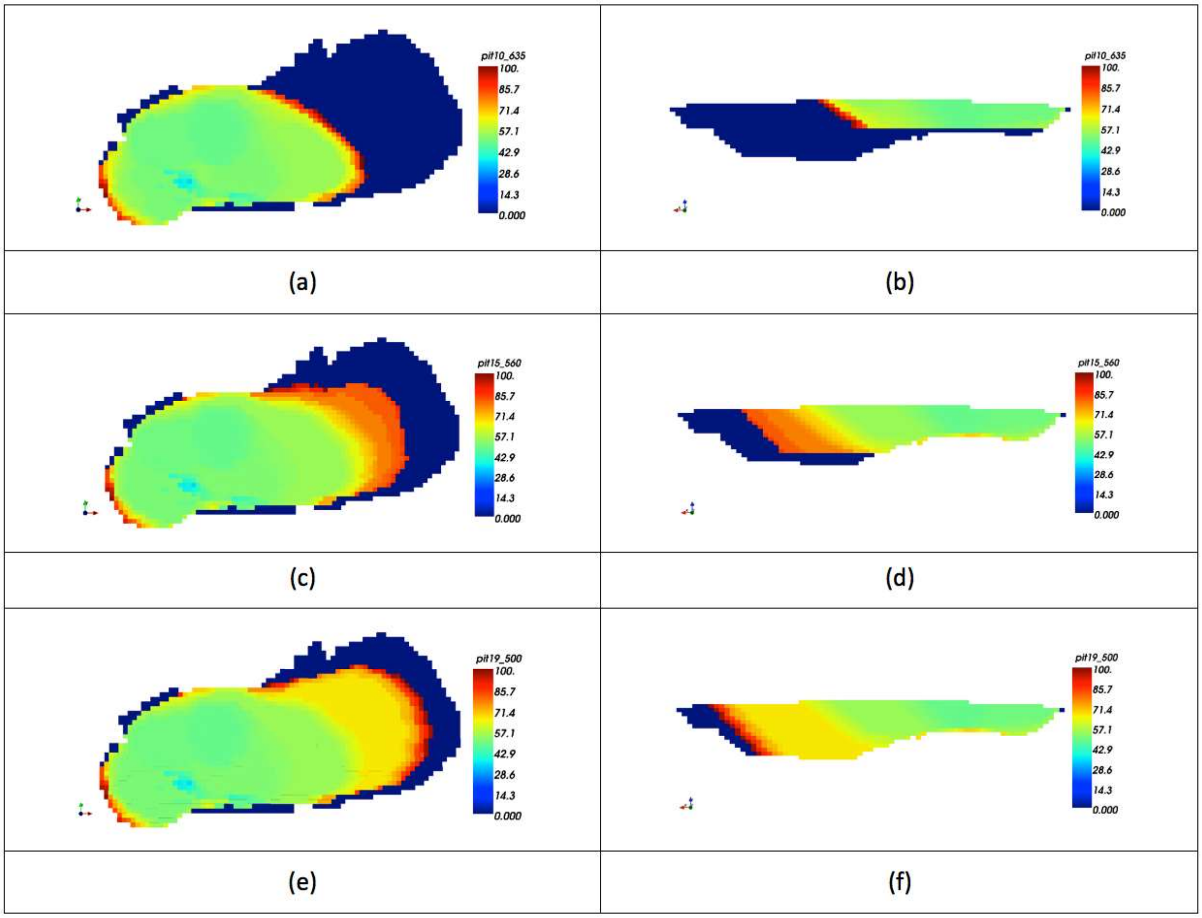

Figure 6 Plant view (left) and cross-section view (right) of intermediate pits generated by RF when depth is limited from top to bottom: (a-b) first 10 benches; (c-d) first 15 benches; (e-f) entire block model

Figure 7 shows the base case result in a pit-by-pit graph (plotting ore and waste tonnages) using the traditional methodology based on the LG algorithm and applying successively a revenue factor over the metal price in the economic block valuation, but considering the entire block model, that is, setting $z^* = z^{min} = 500m$, where z^{min} is the deepest bench so $B(z^* = z^{min}) = B$. Although 100 potential pits were generated, 67 of them were non-empty pits. In this result, there are two large jumps between successive pits (47-48 and 61-62) on the rock tonnage and it is not possible to reduce this difference (gap problem) under a properly chosen set of revenue factors, conditioning the subsequent phase selection task.

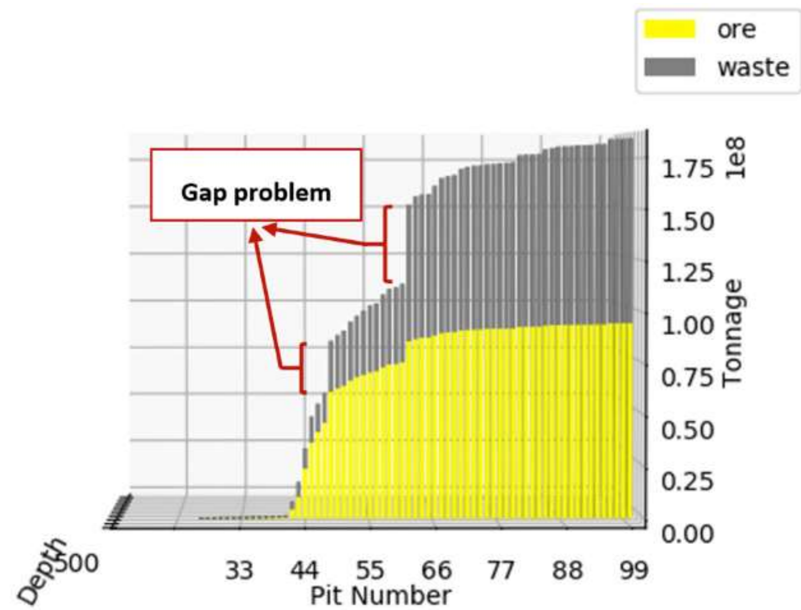

Figure 7 **Ore/waste tonnages vs. revenue factor (pit-by-pit graph)**

Figure 8 shows ore and waste tonnages for each pit obtained by RF and depth. The intermediate pits when considering the first two benches ($z^* = 770m, 765m$) are empty. For other RF and depth choices, there are 1,139 non-empty intermediate pits as compared to the 67 non-empty pits obtained under the traditional methodology, providing many more alternatives for generating the selection of phases. In this case, the planning engineer can make the phase selection by moving from one pit ($RF_1, depth_1$) to another ($RF_2, depth_2$) as long as one of its dimensions keeps growing, i.e. $RF_2 \geq RF_1$ and $depth_2 \geq depth_1$ (depth measured from the surface).

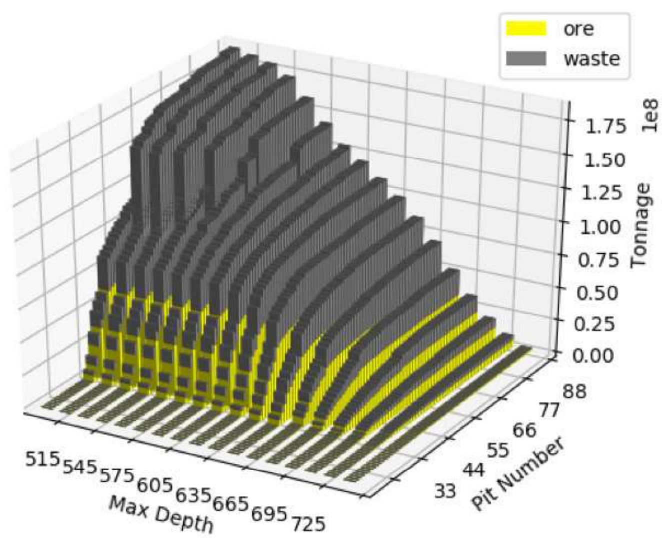

Figure 8 **Ore/waste tonnages vs. (revenue factor x max. depth)**

4.2 Extended intermediate pits through expected extraction periods limiting depth

A completely different approach incorporates the opportunity cost and time value of money into the decision of the pits generated as a guide to the extraction sequence through the concept of the expected extraction period (ET). The results show that an important consideration with respect to Section 4.1 is that the expected extraction periods identify different areas in which to begin the mining sequence. This can be seen in Figure 9, which shows several views of intermediate pits obtained by ET when depth is limited to different values (z^* = $635m$, $560m$, $500m$). In this case, for a given depth, the set of pits generated through ET are nested. Unfortunately, when an ET value is set, the set of pits generated by modifying depth is not nested because, when new exploitation benches are revealed, the nature of the (R-$OPBSP$) model is to search for the portion of those richest blocks found at greater depth as soon as possible, thus changing the distribution of ET values. However, despite losing the property of being nested in the depth direction, new intermediate pits based on ET can help the planning engineer search for new extraction sequences, albeit taking care not to fall into non-feasible solutions.

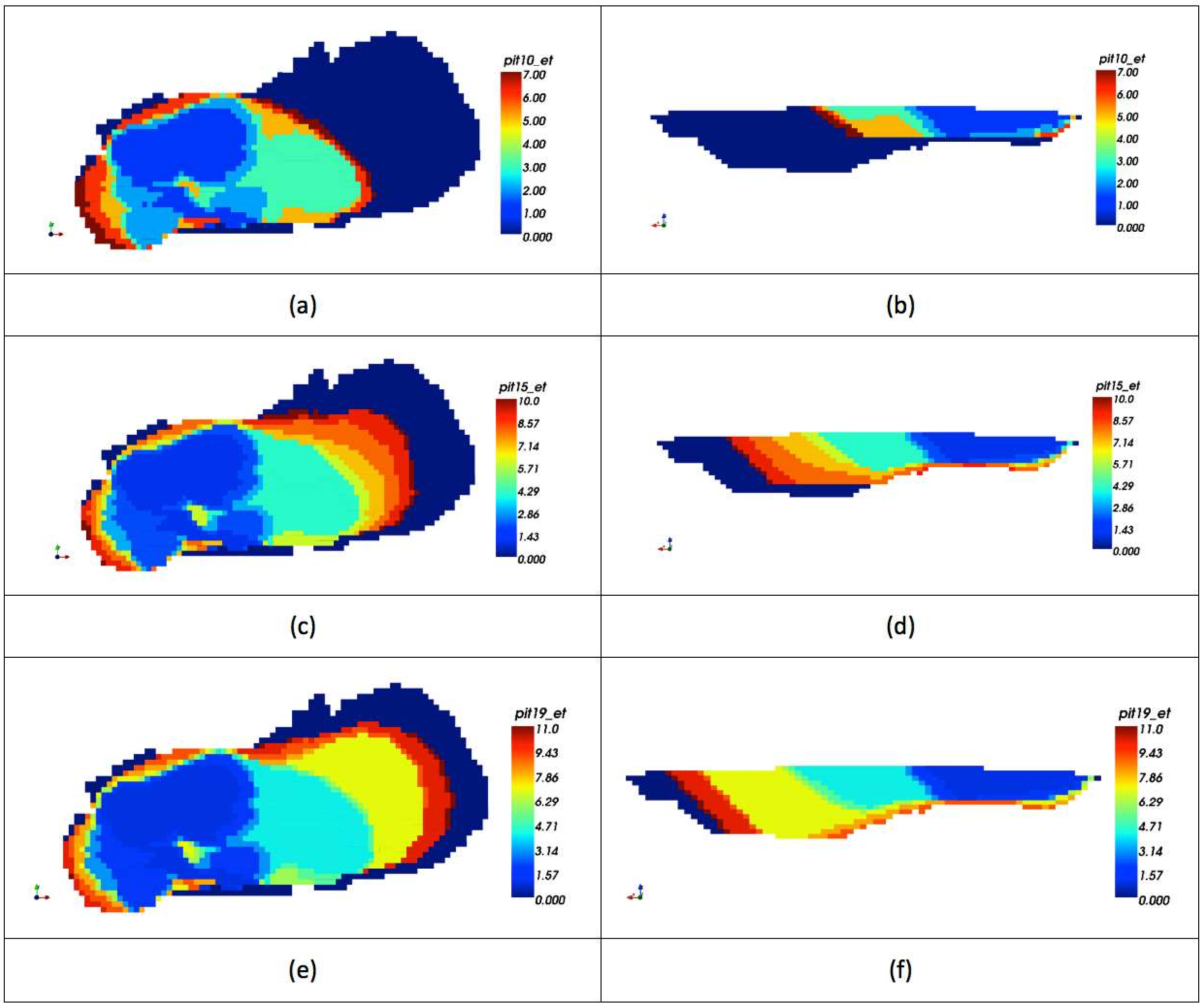

Figure 9 Plant view (left) and cross-section view (right) of intermediate pits generated by ET when depth is limited from top to bottom: (a-b) first 10 benches; (c-d) first 15 benches; (e-f) entire block model

Figure 10 shows ore and waste tonnages for each pit obtained by ET and depth. Again, when considering the first two benches ($z^* = 770m, 765m$), the intermediate pits are empty. The maximum expected extraction period obtained was $ET = 11$ (the mine was scheduled entirely in these 11 periods, leaving period 12 without assigned blocks). For other ET and depth choices, there are 187 non-empty intermediate pits as compared to the 67 non-empty pits obtained under the traditional methodology.

Because ET generation respects capacity constraints on resource consumption, there is a control on rock and ore tonnages, which explains a more constant growth rate for the RF vs. the depth case. However, changes in ET distribution do not generate nested pits. The red ring in Figure 9 highlights an example where the property of being nested is not fulfilled. This results in a non-increasing rock tonnage when advancing in the depth dimension for a given ET.

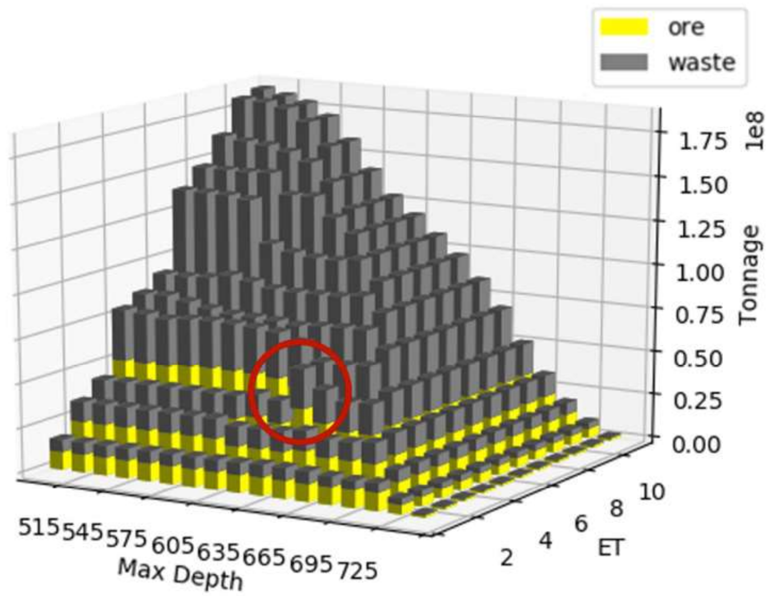

Figure 10 Ore/waste tonnages vs. (expected extraction period x max. depth)

5 Conclusions

This proposed extension of the traditional generation of intermediate pits using RF is based on two ideas: (i) limiting the depth of the pits; and (ii) incorporating the time dimension in the definition of pits. It is intended as a starting point for providing mining engineers with more alternatives for selecting phases to guide the extraction sequence in long-term open-pit mining.

In both cases, the searching space for phase selection is larger than the space offered by the single-dimension RF-based methodology. This has advantages that include (i) it offers options through which the mining engineer can avoid the gap problem; (ii) pits with a limited depth are more appropriate from the operational point of view because of the distances maintained in pit bottoms; and (iii) the opportunity cost over the planning horizon is considered in pit generation. The integration of ET is intended not only to reduce the gap but also to increase the benefit from extracting ore since the discounted block value is considered. However, ET does not generate nested pits when combining with different depths and this disadvantage should be considered in the assessment.

In future work, some challenges should be addressed:

1. Evaluation of the impact of an extended set of intermediate pits in phase generation and the production scheduling stage in terms of, for example, NPV and operational spaces;

2. Proposal of a method that assists the mining engineer in the phase selection task since new information generates more options for the choice of phases and increases its complexity manually.

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