MIP-based procedure to pushback selection

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In long-term open-pit mine production planning, the determination of pushbacks strongly determines the production scheduling strategy for the mine and therefore it is critical for the final design and profit obtained. In order to determine the pushbacks, the standard approach is to rely on a set of ultimate pit problems, usually parameterized by revenue factors to scale the metal price in order to obtain nested pits from where to pick pushbacks. However, this procedure has been shown to be limited in several aspects, like that it does not consider the opportunity cost in production scheduling. Conversely, the alternative approach of direct block scheduling focuses on the optimal timing for block extraction and processing, but does not provide a way to define pushbacks and therefore mining phases.

In this paper, a hybrid approach that aims to make use of the best of the nested pits and the direct scheduling approach. For this, the proposed procedure solves an LP relaxation of a simplified version of direct block scheduling problem and then uses this solution to estimate the optimal extraction time of each block. These estimates are then used to adjust the value of the blocks so that the new values are then used to generate pushbacks by the nested pits approach. The methodology is applied on a case study, showing that it was able to produce alternative sets of pushbacks to choose from and also so that they overcome some of the traditional nested pits limitations, like repeated pits or the gapping problem.

Introduction

The production scheduling of an open-pit mine requires to solve a geometric or space problem related to the volumes (pushbacks) being extracted, and then scheduling the extraction of these volumes over time. Therefore, the determination of these pushbacks is critical for the final design of the mine and the profit obtained.

Traditionally, the pushbacks selection is done manually by expert mine planning engineers, based on the nested pits obtained using the methodology developed by Lerchs and Grossmann (1965). From the total number of generated nested pits, a selected number is used to define the pushbacks, based upon selected criterion (or criteria), for instance, minimum operational width that must be maintained. However, this procedure has important limitations: (i) it does not guarantee that the ore and waste tonnages are uniformly distributed between the pushbacks, which could affect the quality of the scheduling stage (also known as "gap problem"); (ii) the in-situ grade uncertainty is not taken into account; and (iii) generally, the selection of pushbacks is a subjective decision of a mine planning engineer.

In order to determine the pushbacks, the standard approach is to rely on a set of ultimate pit problems, usually parametrized by revenue factors to scale the metal price in order to obtain set nested pits from where to pick the pushbacks. This procedure is known as LG algorithm and it has been shown to be limited in several aspects, one of them being the fact that as the production scheduling is performed in later stages, it is does not consider the opportunity cost and therefore it may lead to suboptimal choices.

Vallet (1976) developed a variant of the LG algorithm that produces a series of nested pits by searching, at each stage, for the pit with the highest revenue/volume ratio among all the feasible pits in the graph. Another variant of this algorithm was developed by Zhao and Kim (1992), where the blocks are aggregated after it has been noted that a block in a profitable group lies under a block in an unprofitable group. Seymour (1995) modified the LG algorithm to incorporate pit volume as a parameter, following the approach of the LG method with the addition of the parametrized variables to produce a series of nested pits. Wang and Sevim (1995) proposed a pushbacks design algorithm imposing an upper bound on the size of the incremental pushbacks in order to overcome the gap problem. Contrary to economic parameterization (metal price, mining cost, cutoff grades, etc), Dagdelen and Francois-Bongarcon (1982) replaced the economic parameters by ore content and recoverable metal quantity. They presented an algorithm that generates a series of nested pits, but parameterizing the metal content and volume of incremental pit. Ramazan and Dagdelen (1998) developed a pushback design algorithm, where among all possible pushbacks with the same size, the algorithm finds the one with minimum stripping ratio. Other authors include floating cones, such as Pana (1965), Wright (1999) and Kakaie et al. (2012) or Lagrangian relaxation methods, for example Dagdelen and Johnson (1986). Somrit and Dagdelen (2013) present a max flow-Lagrangian based phase design algorithm, including time value of money and blending requirements in its formulation. A complete reference for the pushbacks generation can be found in Meagher et al. (2014), where a review of methods are examined in order to produce pushback designs, particularly, how they can tackle the gap problem.

An alternative to the procedure described before is direct block scheduling. In this case, optimization techniques are used in order to define the extraction sequence of the blocks so that, for example, a maximum cumulative discounted value is attained. While this approach is able to include capacity constraints and therefore to consider the opportunity cost, at the moment it is not clear that the solution obtained in this way may allow for operational designs (hence, the spatial component of the problem). Besides this approach has the issue of the computational complexity of solving the mathematical problems, which can be very large. For this reason, many authors have worked on developing schemas to approach variations of this problem, for example, see Johnson (1969) and Caccetta & Hill (2003). For a review, see Osanloo et al. (2008) and Newman et al. (2010).

In this paper, a hybrid approach that it allows to include the opportunity cost for the selection of the pushbacks is proposed. The procedure consists of the following steps: (i) to solve LP relaxation of a simplified version of direct block scheduling problem, (ii) to estimate the extraction time of each block, and (iii) to use the expected extraction times to generate pushbacks.

Methodology

Let B be a block model. For convenience, this set represents a final pit. Each block is denoted with indices b, b', and a number of attributes are given for each block, for example, value, ore grade and tonnage. The block value v_b of block b is given by

$$\boldsymbol{v_b} = \begin{cases} \left[(\boldsymbol{p} - \boldsymbol{c_s}) \cdot \boldsymbol{y_b} \cdot \boldsymbol{rec} - \boldsymbol{c_p} - \boldsymbol{c_m}\right] \cdot \boldsymbol{ton_b} & \boldsymbol{if\ y_b} \geq \dfrac{\boldsymbol{c_p} + \boldsymbol{c_m}}{(\boldsymbol{p} - \boldsymbol{c_s}) \cdot \boldsymbol{rec}} \\ -\boldsymbol{c_m} \cdot \boldsymbol{ton_b} & \boldsymbol{otherwise}. \end{cases} \tag{1}$$

where

p	=	metal price.	rec	=	metallurgical recovery.
c_s	=	selling cost.	y_b	=	ore grade of block b.
c_m	=	mining cost.	ton_b	=	total tonnage of block b.
c_p	=	processing cost.			

In the traditional methodology, a parameter called revenue factor scales metal price in (1) by generating a set of nested pits. However, it is not clear how these factors are chosen, although ideally should be defined in such way that the pits increment as uniform as possible, in terms of volume and tonnage. Unfortunately, it is not always possible to do this (for example due to the *gapping problem*), so in some cases heuristics methods may be used to separate pushbacks. Consider the following notation:

x_{bt}	=	binary variable, 1 if block b is extracted by period t, and 0 otherwise.
t	=	time period. t takes values from 1 to T (T horizon planning).
η	=	discount rate. It represents the opportunity cost.
$oton_b$	=	ore tonnage in block b.
MC_t	=	mining capacity per period t.
PC_t	=	processing capacity per period t.
A	=	set of precedence arcs. $(b, b') \in A$ means that block b has to be extracted before block b'.

In the proposed methodology, the aim is to use temporary information in order to generate a set of pushbacks: an approximation of the extraction period of each block is used to decide it. For this purpose, it considers the following version of the production scheduling problem, known as *constrained pit limit problem* (CPIT) (Chicoisne et al. 2012), which maximizes the net present value of extracted blocks.

$$(CPIT) \quad \max \sum_{b\in B}\sum_{t=1}^{T} \frac{1}{(1+\eta)^t} \cdot v_b \cdot (x_{bt} - x_{b,t-1}) \tag{2}$$

$$\text{s.t.} \quad x_{b't} \le x_{bt} \qquad \forall\, (b,b') \in A, t = 1, \dots, T \tag{3}$$

$$x_{bt} \ge x_{b,t-1} \qquad \forall\, b \in B, t = 1, \dots, T \tag{4}$$

$$\sum_{b\in B} ton_b \cdot (x_{bt} - x_{b,t-1}) \ \le MC_t \qquad \forall\, t = 1, \dots, T \tag{5}$$

$$\sum_{b\in B} oton_b \cdot (x_{bt} - x_{b,t-1}) \le PC_t \qquad \forall\, t = 1, \dots, T \tag{6}$$

$$x_{bt} \in \{0,1\}, x_{b0} = 0 \qquad \forall\, b \in B, t = 1, \dots, T \tag{7}$$

Expression (2) presents the objective function, which is the cumulative discounted value of extracted blocks over T. In turn, (3) corresponds to the precedence constraints given by the slope angle and (4) means that blocks can be extracted only once. Moreover, (5) and (6) state the maximum resource consumption in each period (mining and processing), respectively. Finally, (7) states that all the variables assume binary values. In summary, the solution of this problem represents what blocks are mined and when, while cumulative discounted value is maximized, subject to precedence between blocks and operational resource constraints. It has been reported that (CPIT) problem is strongly NP-hard (Johnson and Niemi 1983). However, there exist good improvements to solve its linear relaxation, for instance, using the BZ algorithm (Bienstock and Zuckerberg 2010). Solving the linear relaxation of (CPIT) using BZ algorithm is computationally efficient and fast, allowing to solve instances for which another algorithms cannot.

Let $\boldsymbol{x}^* = \{x_{bt}^*\}$ be the optimal solution of the relaxed version of (CPIT) problem. Then, the expected time of extraction ET_b for block b is given by

$$ET_b = \sum_{t=1}^{T} t \cdot (x_{bt}^* - x_{b,t-1}^*) + (T+1)(1 - x_{bT}^*) \qquad \forall b \in B \tag{8}$$

If $\boldsymbol{x}^*$ is the optimal integer solution of (CPIT), then ET_b represents exactly the period of extraction of block b. Note that non-scheduled blocks are assigned to fictitious extraction period $T+1$. This classification prompts a partition of B, where each subset j has the same expected time ET^j. A nested pits-based interpretation is possible if it considers the j^{th} pit as the subset of blocks with expected time lower than ET^j.

Now, the question is how to use this new information to generate pushbacks. Given that expected times identify groups of blocks satisfying resources capacities and maximizing discounted value, besides that $(b,b') \in A$ imply $ET_b \le ET_{b'}$, a proposal is to define the pushbacks P_k as

$$P_k = \left\{ b \in B \colon \frac{(k-1)T}{n_o} < ET_b \le \frac{kT}{n_o} \right\} \qquad \forall\, k = 1, \dots, n_o \tag{9}$$

where n_o is the number of desired pushbacks. Regarding to traditional methodology, parameterizing metal price into (1) by revenue factors λ_i is obtained

$$\boldsymbol{v_b^i} = \begin{cases} \left[(\boldsymbol{p} \cdot \boldsymbol{\lambda_i} - \boldsymbol{c_s}) \cdot \boldsymbol{y_b} \cdot \boldsymbol{rec} - \boldsymbol{c_p} - \boldsymbol{c_m}\right] \cdot \boldsymbol{ton_b} & \boldsymbol{if\ y_b} \ge \dfrac{\boldsymbol{c_p} + \boldsymbol{c_m}}{(\boldsymbol{p} \cdot \boldsymbol{\lambda_i} - \boldsymbol{c_s}) \cdot \boldsymbol{rec}} \\ -\,\boldsymbol{c_m} \cdot \boldsymbol{ton_b} & \boldsymbol{otherwise}. \end{cases} \tag{10}$$

Using (10) leads to the final pit type problems obtaining a family of nested pit (Equations 11 through 13):

$$(FP_i) \quad max \sum_{b \in B} v_b^i \cdot x_b \tag{11}$$

$$\text{s.t.} \quad x_{b'} \leq x_b \qquad \forall\, (b, b') \in A \tag{12}$$

$$x_b \in \{0,1\} \qquad \forall\, b \in B \tag{13}$$

where x_b is a binary variable for final pit (in this case, for intermediate pit) selection. There exist good algorithms to solve quickly this problem (Chandran and Hochbaum, 2009). In order to select n_o pushbacks from nested pits, the methodology proposed in Jélvez et al. (2016) may be used, where it was presented a model to automate pushback selection.

Finally, in order to make a discounted value analysis, the extraction shall be done from the first pushback through the last one and from the most upper bench through the lowest one, respecting operational resource capacities imposed by (5) and (6), following a phase-to-phase extraction strategy.

Case study

In order to compare the proposed methodology against the traditional one, a case study shall be presented. The dataset, named Arizona's Copper Deposit (KD), was obtained from Minelib (Espinoza et al. 2013), a publicly library of open-pit mining problems available in http://mansci-web.uai.cl/minelib/kd.xhtml. The final pit contains 185 [Mton] of rock with 95 [Mton] of mineralized material. The parameters used to define a scheduling instance for (CPIT) problem are presented in Table 1. For a given block, the precedence arcs include all blocks inside a cone defined by a slope angle 45° and height given by 8 benches on it.

Symbol	Value	Unit
p	2.5	[USD/lb]
c_s	0.4	[USD/lb]
c_m	3.2	[USD/ton]
c_p	9.0	[USD/ton]
rec	0.9	
η	0.15	
T	10	[year]
MC_t	40	[Mton]
PC_t	10	[Mton]

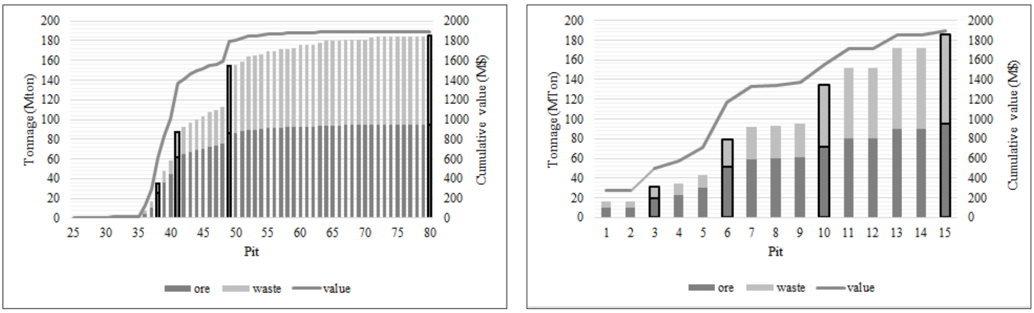
Table 1: List of economic and technical parameters.

On the one hand, based upon traditional methodology, hereinafter called (M1) as well, a series of 80 revenue factors was constructed from $\lambda_i = \text{i}/80$, where $i = 1, \ldots, 80$. Then, a set of nested pits is found repeatedly solving (FP_i) by using (11) - (13) and a customized version of pseudoflow algorithm (MineLink, 2013), parameterizing the block value through (10) and from which $n_o = 4$ pits are selected as pushbacks. On the other hand, the relaxed version of (CPIT) is solved using a customized version of BZ algorithm (MineLink, 2013) considering an optimality gap of 1%. MineLink is a library of data structures for mining scheduling problems and algorithms to solve them, by providing a set of tools and well-stated problems to work on. Then, the whole mine is partitioned into groups according to expected times, and a set of pushbacks are computed by using (8) and (9) with $T = 10$ and $n_o = 4$. This methodology shall also be called (M2) for short.

The experiments were executed on an Intel Core i7-4510U, 8 Gb memory ram and 4 cores to 3.1 GHz machine on Windows 8.1 environment. The optimization software used was Gurobi, v5.6.3, academic license.

Results and discussion

In this section, the main results are shown. For one side, according traditional methodology, 80 revenue factors were arbitrarily chosen from 1/80 to 1, but only from the 26/100 one, a first pit was obtained, therefore 55 nested pits were produced (Figure 1-a), employing 50 seconds. Pit by pit graph shows the tonnages (ore and waste) for each pit and the cumulative undiscounted value. Traditionally, this graph is used to identify candidates for pushback. Then four pushbacks were selected (pits 38-41-49-80), taking care suitable distribution of ore and waste tonnages. For other side, by using the expected extraction times of blocks, 15 nested pits were generated (pit 15 is the ultimate final pit) and applying (9), pits 3-6-10-15 were selected as pushbacks (Figure 1-b). The list of different expected extraction times are orderly shown in Table 2: the highest expected value ($ET = 11$) it corresponds to non-extracted blocks. The time employed to get expected extraction times was 20 seconds. Note that the growth rate of ore tonnage per pit is relatively constant in the case of pits obtained by the proposed method M2. The other pits (from M1) suffer gap problem in rock tonnage contained in pits 40-41 and 48-49, makes it difficult to find pushbacks with the same tonnage of rock.



(a) (b)

Figure 1: Pit by pit graph: (a) traditional methodology (M1), and (b) expected time extraction-based methodology (M2) proposed in this work. Thick border columns represent pits that are selected as pushbacks.

1.00	1.34	2.00	2.98	3.00	4.60	6.00	6.28
7.00	7.15	8.00	8.56	9.00	9.46	10.00	11.00

Table 2: Different expected values of extraction obtained from relaxed solution of (CPIT).

Table 3 shows a summary of generated pushbacks: for each methodology (M1 and M2), ore and waste tonnages are presented together with associated stripping ratio (SR) and average grades. M2 presented higher SR in the first and second pushback regarding to M1. The following pushbacks presented lower SR than those ones obtained with M1. Bold numbers represent the best case of stripping ratio and average grade for each case. Additionally, Figure 2 shows plant and section views from pushbacks obtained according to M1 (a)-(b), and from pushbacks obtained with M2 (c)-(d). White blocks (code 0) represent blocks outside of ultimate final pit. Note the difference between the pushbacks generated using the traditional and the proposed approaches: the first ones show better shape in the first pushback, because the respective proposed one presents a hole, being more impractical for scheduling, Clearly, the LP version of (CPIT) tries to postpone the extraction of low value/waste blocks, creating this kind of undesirable non-operational mining configurations. An immediate solution is to extend the rule given in (9), modifying the bounds of respective intervals to include these blocks into the first pushback, although that shall reduce the cumulative value, but it should help to improve the operational feasibility.

Pushback	Ore (MTon)		Waste (MTon)		Stripping ratio		Average grade (% Cu)	
	M1	M2	M1	M2	M1	M2	M1	M2
1	25.2	19.9	9.8	11.2	**0.39**	0.56	0.91	**0.94**
2	36.9	31.6	15.6	17.0	**0.42**	0.54	0.81	**0.84**
3	24.1	20.0	42.3	34.6	1.76	**1.74**	0.86	**0.90**
4	9.0	23.7	22.4	27.3	2.49	**1.15**	**0.74**	0.72
TOTAL	95.2		90.1		0.95		0.84	

Table 3: Summary pushbacks selection by means of two methodologies: M1 refers to traditional one and M2 refers to expected time extraction-based one.

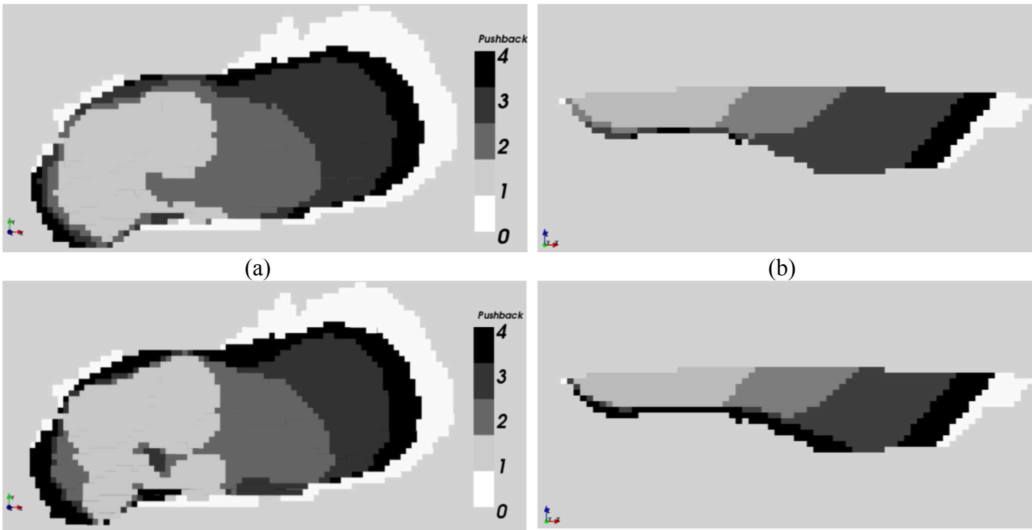


Figure 2: Pushbacks obtained from traditional methodology (M1): (a) plan view and (b) Y=430m section view. Pushbacks obtained from proposed method (M2): (c) plan view and (d) Y=430m section view.

An important aspect in pushbacks design is their role in the production scheduling stage. Pushbacks strongly influence how accessing to the ore and waste blocks in all periods in order to maximize the cumulative discounted value. Figure 3 shows cross-section views of the block schedule obtained from the integer formulation of (CPIT) following a phase-to-phase extraction strategy and imposing that all blocks within final pit must be extracted. Time employed to find each schedule was near to 2.0 hours. Some differences assigning blocks to extraction period can be seen, for instance, the opening of a given pushback starts at different periods when comparing if it comes from M1 or M2: while (first to fourth) pushbacks from M1 are opened at periods 1, 3, 7 and 10, pushbacks from M2 are opened at periods 1, 2, 6 and 8, respectively. This shall cause differences in discounted value obtained per period as well.

Finally, Figure 4 shows a comparison of the production plans (left vertical axis) and cumulative discounted values (right vertical axis) for each methodology M1 and M2: some important comments (i) slight differences are reported between ore columns, but regarding to waste tonnages they have large differences at same periods; (ii) in terms of cumulative discounted value (CDV), the proposed method shows a ~2% higher CDV regarding to the traditional based one, and looking to the values period by period, method M2 shows a higher value regarding to M1 in all periods, but the ninth one. CDV from M1 was 990.1M$ while CDV from M2 was 1,009.5M$.

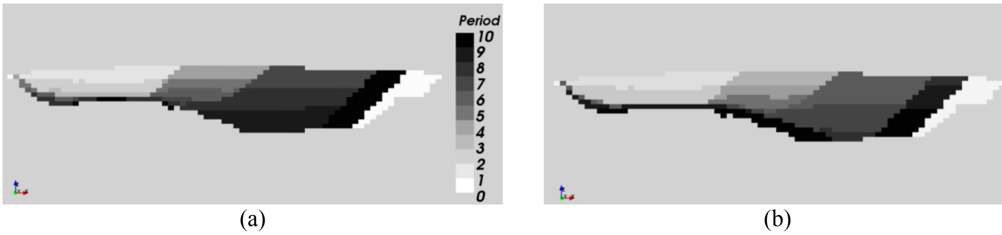


Figure 3: Main results of block scheduling from pushbacks selection: (a) Y=430m cross-section view obtained from M1; (b) Y=430m cross-section view obtained from M2.

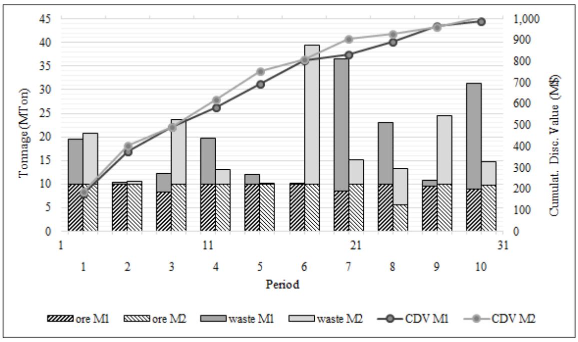


Figure 4: Comparison of production plans and cumulative discounted values for each methodology.

Conclusions

In this work a methodology based on a hybrid approach that includes temporary information in the pushbacks selection is developed. The procedure consists of solving the LP relaxation of a simplified version of direct block scheduling problem, named (CPIT), then estimates the extraction time of each block, with which generates pushbacks.

Contrary to the traditional method using parameterization of metal price, the methodology developed in this work allows the mine planner to control the ore and waste tonnage in each pushback. The method assigns blocks to pushbacks while respecting maximum ore and rock tonnage as well as slope constraints. Besides, it overcomes some of the traditional nested pits limitations, like repeated pits and it is linked to scheduling stage. The main advantage of this method is the incorporation of temporal dimension to the pushbacks selection procedure by means of an approximation of the time of extraction of different portions of the mine, and at time to produce an alternative partition of the ultimate final pit that identify the most valuable sectors, in term of discounted value, unlike the traditional methodology, which does not considers the opportunity cost in its determination. Another advantage of this methodology is that it gives well-defined criteria to select pushbacks from nested pits; therefore, it allows the mine planner to automate this task.

The methodology was applied on a case study, showing that it was able to produce alternative sets of pushbacks with a potential of ~2% higher cumulative discounted value, in comparison to pushbacks from the traditional methodology based on parameterization by revenues, when both set of pushbacks are scheduled according to phase-to-phase extraction strategy.

Fast algorithms have been reported in the literature to support this methodology, such as BZ algorithm. Indeed, this allows to consider as future work, more general problems, such as multiple destinations and blending constraints (similar to work of Somrit and Dagdelen, 2013) as part of pushbacks selection to ensure more complicated operational conditions. For example, the case study shown the necessity to include minimum requirement of capacity constraints, especially associated to mining capacity in order to avoid large variability of rock movement between periods in the scheduling stage.

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