A Comparison of Conventional and Direct Block Scheduling Methods for Open Pit Mine Production Scheduling

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ABSTRACT: Open pit mine production scheduling for long-term planning is a relevant and required task for any mining project or operation. Mining blocks must be scheduled for extraction over a set of years, and a destination must be assigned to each one of them. The goal is to maximize the Net Present Value of the project, subject to capacity and operational constraints. Traditionally, this task has been performed either with the guidance of nested pits produced by the Lerchs-Grossmann algorithm (LG), considering pre-defined block destinations, or by Direct Block Scheduling (DBS), in which individual blocks are selected (or not) for extraction and destinations are assigned at given periods of time.

On the one hand, from the purely theoretical side, DBS methods should be superior to those based on LG, because they are designed to deal with more realistic considerations of the problem (like capacities, multiple products, etc.) while LG approaches are limited to slope constraints and a unique economic value as parameters. On the other hand, the practical one, LG-based methods have been at the advantage, because DBS methods require intensive computational power to be solved.

Fortunately, in later years, the availability of new algorithms and technology has made DBS more competitive. New DBS algorithms based on Integer Programming and heuristics have arisen with reasonable processing times, and MineLib, a set of standard datasets for testing, has been published and made available for researchers and software developers.

This paper presents two DBS algorithms and show, by means of MineLib, their competitiveness against the state-of-the-art algorithms commercially available. Furthermore, these algorithms are applied to a case study in order to see how the solutions obtained differ and improve on the traditional approach based on LG.

INTRODUCTION

A very important task in the mine planning process is the long-term schedule, in which mining blocks have to be scheduled for extraction over a set of years, and a destination must be assigned to each one of them. This schedule is subject to different operational and economical contraints

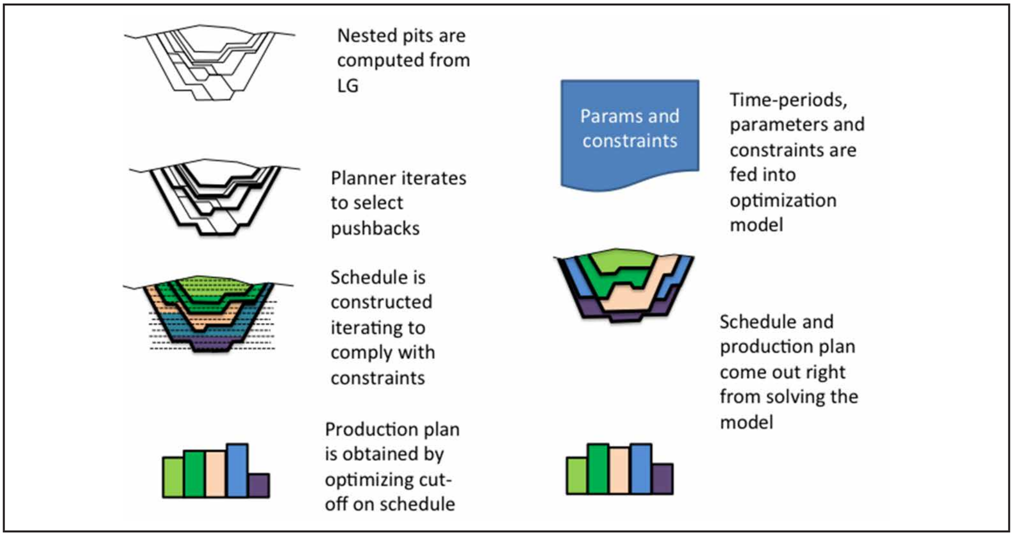

Figure 1. Traditional methodology versus direct block scheduling

(processing capacity, slope angles, etc.) and is oriented to optimize the value, often expressed in terms of the Net Present Value (NPV) of the project.

There are two main approaches for this task. The first one is based on the construction of Nested Pits by means of the Lerchs-Grossmann algorithm (LG) (Lerchs & Grossman 1965). This method relies on the usage of a revenue factor that modulates the pits sizes by penalizing the price in the block valuation, followed by other algorithms for scheduling, for example, Millawa and cutoff optimization (Gemcom 2011).

The second approach is based on direct block scheduling (DBS), that is to directly assign extraction periods to the blocks by means of an underlying mathematical optimization problem (Johnson, 1968). While this approach is theoretically better, it has the issue of the computational complexity of solving the mathematical problems, which can be very large. For this reason, many authors have worked on developing schemas to approach variations of this problem (see Caccetta and Hill 2003; Chicoisne et al. 2012; Bienstock and Zuckerberg 2010 for some examples, and Newman et al. 2010 for a review).

Fortunately, in later years, the availability of new algorithms and computational technology has made DBS less time consuming and, therefore, more competitive. This, in turn, has motivated more research in the area and, for example, currently there has been published MineLib (Espinoza et al., 2013), a public library of case studies on which it is possible to compare different method for DBS.

Figure 1 briefly compares the two main methodologies for mine scheduling. On the left, the conventional method that relies on LG to generate nested pits which are used as a basis to select pushbacks and then schedule the production. On the right, the DBS method, in which all parameters and constraints are fed into an optimization model that which solutions is schedule and a production plan. It is important to notice that, for the conventional method, compliance with the constraints is something that the planner must ensure, while the optimization models cannot produce solutions that violate a constraint.

This paper presents two DBS algorithms and show, on a subset of MineLib instances, their competitiveness against the published results for these cases. Furthermore, these algorithms are applied to a case study in order to see how the solutions obtained differ to the traditional approach based on LG.

Conventional Method for Production Scheduling (Whittle)

Very briefly, the conventional method based on Lerchs and Grossman algorithms can be expressed as follows

1. Compute nested pits using Lerchs and Grossman, by parameterizing the block values using a revenue factor.
2. Select pushbacks from nested pits.
3. Generate a production schedule using blocks in each phase/pushback.
4. Improve the production schedule by optimizing the cut-off grades.

It is worth noting that only steps 1 and 3 are based on optimization models or known algorithms, but the pushback selection is made by the planner, with the goal of generating a reasonable production plan that integrates relevant constraints like mine and plant capacity. The production schedule is generated using some general algorithms that, at the end, can be translated into block schedules. Finally, step 4 can be assisted by optimization tools, but it mainly depends on the ability of the planner to make the best choices. Notice we are not dealing here with the design of pit phases, which are also strongly user dependent.

Direct Block Scheduling for Production Scheduling

DBS follows a different approach that aims to integrate all the steps above, so the pushbacks already comply with some constraints, like total or plant capacity. This approach is based on mathematical programming and ad-hoc algorithms to solve them.

SimSched

SimSched is a commercial software developed by MiningMath that does DBS. For this, and while SimSched does rely on mathematical optimization to compute the schedules, it does not rely on precedence arcs to represent constraints related to slope walls. Instead, SimSched works directly with surfaces. On the good side, this allows integrating operational constraints, such as a minimum bottom width, vertical advance rates, and user-defined physical limits. On the bad side, the schedules generated are not fully compliant with MineLib instances because the schedules are expressed as surfaces and not block extraction periods. This is a theoretical issue that is not important for the scope of this paper.

The exact model of SimSched is undisclosed, but closer ones can be found in Goodwin et al. (2005) and Marinho (2013).

MineLink and BOS2

MineLink is a set of optimization routines and models developed at the Universidad de Chile, with the aim to provide general and easy to use tools for teaching and research in mine planning area. One of the modules developed within this framework, is the "Blending Optimization Sequencing and Scheduling" (BOS2), a set of routines for DBS (See, for example, Jélvez 2012).

Table 1. Test Instances from MineLib

Instance Name	Description	#Blocks	Capacity Constraints
MARVIN	A synthetic block model from GEMCOM Whittle software. It contains copper and gold grades. Block sizes are 30x30x30 meters.	53,271	Plant Capacity 20Mtons/period Mining Capacity 60Mtons/period
KD	A copper block model from a mine in Arizona, with 1 waste and 2 process destinations. Block sizes are 20x20x15 meters.	14,153	Plant Capacity 10Mtons/period No Mining Capacity Limit
McLaughlin	A gold mine from CA, USA. Block model contains gold grades. Block sizes are 25x25x20 feets.	2,140,342	Plant Capacity 3.3Mtons/period No Mining Capacity Limit

METHODOLOGY

The methodology is straightforward:

1. Select suitable instances from MineLib for usage and comparison.
2. Execute SimSched and BOS2 on these instances to generate production plans and pit profiles.
3. Use GEMCOM Whittle on the instances to generate production plans and pit profiles.
4. Compare the results.

CASE STUDIES

We have taken the case studies from MineLib, which is a set of mine scheduling problems that can be used objectively for comparison and algorithms. This library contains 11 instances for two specific problem types: CPIT, with fixed block destinations, and PCPSP, in which block destinations can be chosen in the optimization process.

For this paper, we have worked with PCPSP cases and selected the instances reported in Table 1. Other instances could not be used because: they do not provide enough geometric or geological information required by Whittle and SimSched; they have blending constraints (not implemented in SimSched yet), or were not interesting (the only case: *McLaughlin_limit*, which is a subset of *McLaughin*).

Also, among these instances, only McLaughin was run using Whittle. This is because KD is too small and MARVIN comes as part of this software.

RESULTS AND ANALYSIS

Production Plans

The columns presented in each table are the following

- **Period:** time-period in the production plan, in years.
- **Prod:** ore production for the given time-period, in millions of tonnes.
- **Waste:** waste production for the given time-period, in millions of tonnes.
- **Grade:** average grade of main mineral for the corresponding time-period. This can be percentage or oz/ton.
- **Disc. value:** discounted cash flow of the given time-period, in millions of dollars.

Marvin

As reported in Table 2 and Figure 2, we observe that SimSched found the highest NPV solution of all three methods, followed by BOS2, and that both are better than the published solution in MineLib. The "bad" performance of MineLib is due to the large waste extraction in period 1, which is probably a consequence its algorithm aims to saturate all capacities.

Table 2. Production plans for Marvin case study

Period	SimSched				BOS2				MineLib			
	Prod. (Mt)	Waste (Mt)	Grade %	Disc. value (M$)	Prod. (Mt)	Waste (Mt)	Grade %	Disc. value (M$)	Prod. (Mt)	Waste (Mt)	Grade %	Disc. value (M$)
1	20.1	14.6	0.44	126.1	20.0	15.4	0.43	126.0	20.0	40.0	0.34	74.1
2	20.1	9.0	0.65	139.6	20.0	9.6	0.65	132.7	20.0	12.8	0.53	112.2
3	20.1	9.7	0.69	123.4	20.0	11.6	0.66	124.4	20.0	6.9	0.62	111.8
4	20.2	12.6	0.70	109.4	20.0	12.3	0.72	108.8	20.0	2.1	0.76	134.2
5	20.1	7.8	0.70	96.1	20.0	11.1	0.69	93.5	20.0	1.5	0.87	126.8
6	20.1	15.7	0.71	79.5	20.0	18.2	0.74	80.3	20.0	13.3	0.74	85.5
7	20.0	15.0	0.66	67.1	20.0	16.5	0.66	64.3	20.0	23.2	0.66	60.6
8	20.2	17.1	0.66	55.9	20.0	17.1	0.68	52.6	20.0	11.2	0.66	57.9
9	20.2	21.6	0.64	43.4	20.0	22.7	0.61	42.1	20.0	23.0	0.62	41.7
10	20.0	20.1	0.58	33.5	20.0	22.0	0.59	33.5	20.0	29.2	0.60	31.0
11	20.1	30.7	0.57	23.3	19.9	27.9	0.56	24.2	20.0	20.6	0.56	26.6
12	20.1	33.5	0.52	14.8	19.9	29.5	0.51	15.6	20.0	34.2	0.52	14.6
13	20.1	29.1	0.47	8.9	20.0	39.1	0.47	7.2	20.0	34.5	0.48	8.7
14	4.8	8.8	0.44	0.4	5.3	9.3	0.45	0.5	5.5	9.3	0.44	0.8
TOTAL	**266.3**	**245.3**	**0.61**	**921.3**	**264.9**	**262.2**	**0.61**	**905.8**	**265.5**	**261.7**	**0.61**	**886.3**

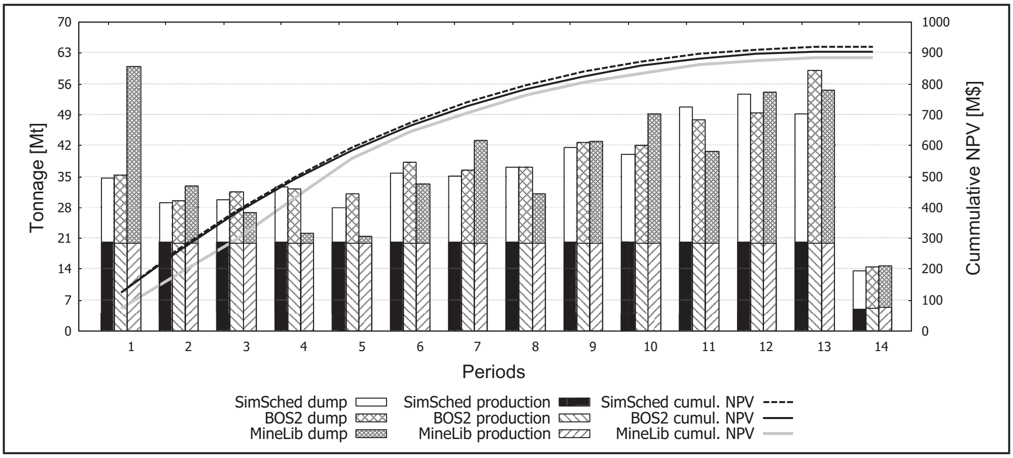

Figure 2. Production plans for Marvin case study

KD

In this case, we observe that BOS2 and MineLib generate solutions with a time-span shorter by 2 years than the one in SimSched. Again, there is no noticeable difference in the NPV of the production plans obtained, but BOS2 produces the solution with the highest NPV (and therefore the best known solution for this instance so far).

Table 3. Production plans for KD case study

Period	SimSched				BOS2				MineLib			
	Prod. (Mt)	Waste (Mt)	Grade %	Disc. value (M$)	Prod. (Mt)	Waste (Mt)	Grade %	Disc. value (M$)	Prod. (Mt)	Waste (Mt)	Grade %	Disc. value (M$)
1	10.0	15.6	1.04	89.2	10.0	7.8	1.02	93.9	10.0	7.7	1.02	93.6
2	10.0	25.5	0.98	63.5	10.0	12.3	0.94	68.0	10.0	12.4	0.92	66.0
3	10.0	10.2	0.92	58.4	10.0	2.5	0.84	55.6	10.0	2.8	0.85	57.0
4	10.0	3.5	0.82	45.7	10.0	8.9	0.87	46.3	10.0	10.3	0.81	41.0
5	9.9	2.8	0.85	42.6	10.0	2.8	0.85	42.3	10.0	34.6	0.86	29.4
6	10.0	1.9	0.83	36.3	10.0	31.2	0.99	34.8	10.0	0.1	0.95	44.0
7	10.0	3.0	0.79	29.5	10.0	8.1	0.83	29.6	10.0	3.8	0.89	34.2
8	10.0	4.5	0.73	22.3	10.0	5.2	0.75	22.8	10.0	5.5	0.77	23.4
9	10.0	5.5	0.60	14.0	10.0	12.9	0.68	15.0	10.0	13.0	0.70	15.7
10	10.0	0.0	0.20	3.3	8.9	0.0	0.16	1.5	3.8	6.8	0.55	2.6
11	10.0	0.0	0.15	1.7	0.0	0.0	0.00	0.0	0.0	0.0	0.00	0.0
12	5.3	0.0	0.10	0.2	0.0	0.0	0.00	0.0	0.0	0.0	0.00	0.0
TOTAL	**115.2**	**72.5**	**0.69**	**406.6**	**98.9**	**91.6**	**0.80**	**409.7**	**93.7**	**96.8**	**0.85**	**407.0**

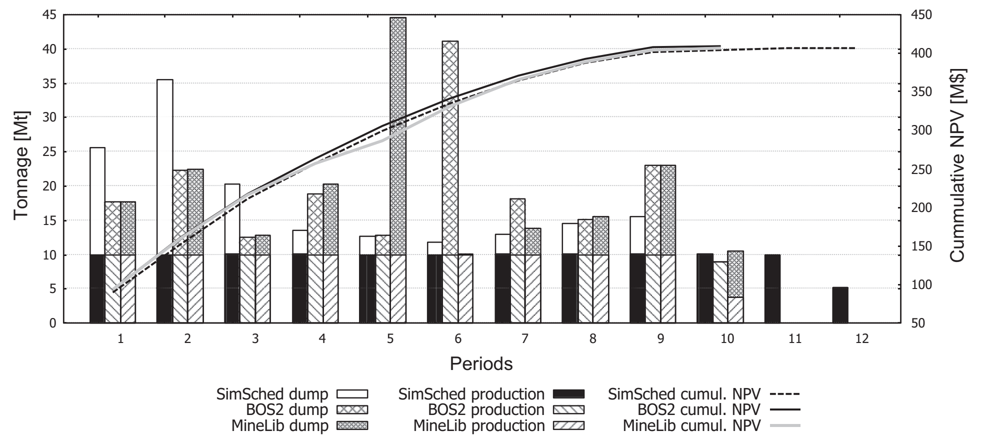

Figure 3. Production plans for KD case study

McLaughlin

In the case of McLaughlin, we include runs from Whittle in the comparison. As before, it is interesting to notice that the final NPV differences are not very big. BOS2 and the solution published in MineLib seem to have a little advantage.

Table 4. Production plans for McLaughlin case study

Period	SimSched				BOS2				MineLib				Whittle			
	Prod. (Mt)	Waste (Mt)	Grade oz/ton	Disc. value (M$)	Prod. (Mt)	Waste (Mt)	Grade oz/ton	Disc. value (M$)	Prod. (Mt)	Waste (Mt)	Grade oz/ton	Disc. value (M$)	Prod. (Mt)	Waste (Mt)	Grade oz/ton	Disc. value (M$)
1	3.3	29.5	0.22	485.4	3.3	14.7	0.22	490.6	3.3	16.2	0.21	478.7	3.3	6.4	0.19	471.4
2	3.3	10.3	0.16	306.0	3.3	13.4	0.16	303.4	3.3	11.9	0.16	310.8	3.3	12.6	0.24	291.1
3	3.3	11.1	0.13	203.4	3.3	15.6	0.13	204.9	3.3	15.5	0.13	206.0	3.3	10.2	0.16	195.0
4	3.3	10.0	0.11	141.5	3.3	11.3	0.12	149.1	3.3	11.3	0.12	149.9	3.3	9.5	0.15	162.5
5	3.3	15.4	0.09	82.6	3.3	7.4	0.10	101.8	3.3	7.3	0.10	101.6	3.3	8.3	0.11	102.9
6	3.3	4.5	0.08	73.3	3.3	12.4	0.09	78.7	3.3	6.1	0.08	72.8	3.3	16.2	0.16	71.1
7	3.3	3.2	0.08	56.0	3.3	2.1	0.07	51.6	3.3	8.6	0.07	48.0	3.3	9.6	0.09	53.7
8	3.3	3.3	0.07	42.2	3.3	3.1	0.06	38.4	3.3	4.8	0.08	49.3	3.3	4.6	0.06	55.5
9	3.3	3.8	0.06	31.4	3.3	3.6	0.06	29.2	3.3	3.3	0.06	29.4	3.3	5.1	0.05	30.2
10	3.3	2.5	0.06	22.7	3.3	5.2	0.06	21.3	3.3	4.3	0.06	22.4	3.3	6.9	0.05	21.0
11	3.3	2.5	0.05	17.6	3.3	4.6	0.05	15.8	3.3	4.3	0.05	15.6	3.3	5.1	0.04	15.7
12	3.3	3.7	0.05	13.0	3.3	15.8	0.05	11.3	3.3	16.0	0.05	11.3	3.3	5.5	0.04	9.7
13	3.3	8.1	0.05	8.2	3.3	3.9	0.04	7.4	3.3	4.3	0.04	7.2	3.3	14.9	0.08	6.8
14	3.3	11.4	0.04	5.0	3.3	5.1	0.04	4.3	3.3	4.7	0.04	4.3	3.3	6.1	0.03	3.5
15	3.3	7.9	0.04	2.4	3.3	7.6	0.03	2.2	3.3	7.3	0.03	2.4	3.3	10.3	0.04	1.3
16	1.8	7.3	0.04	2.1	2.4	4.8	0.03	0.5	2.4	4.7	0.03	0.5	0.6	1.3	0.03	0.1
TOTAL	**51.3**	**134.5**	**0.08**	**1493**	**51.9**	**130.6**	**0.08**	**1510**	**51.9**	**130.7**	**0.08**	**1510**	**50.1**	**132.5**	**0.10**	**1492**

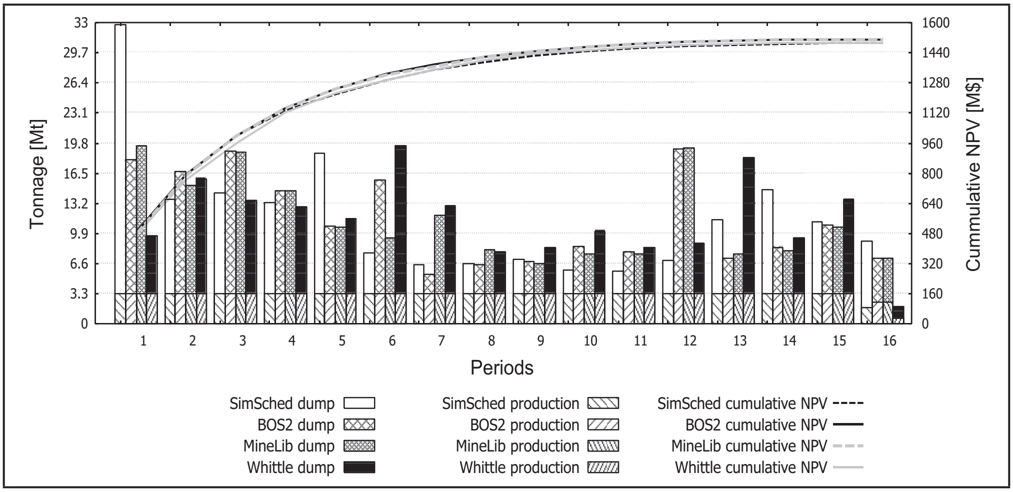

Figure 4. Production plans for McLaughlin case study

Pit Profiles

In this section, we present some pit profiles for the schedules reported before. The profiles follow the deepest block centers (for MineLib, Whittle and BOS2) and the surface reported by SimSched. It is interesting to notice that, contrarily to NPVs, the pit profiles of the pits are quite distinct from each other, especially in the first periods. A possible explanation for this is that the block models used may be a bit homogeneous. Indeed, there are different ways to reach the high NPV schedules (Figures 5–13).

CONCLUSIONS

We have compared the pits and production plans obtained from different methodologies: classical nested pits from LG plus scheduling, using Whittle and DBS.

First of all, it is interesting to notice that while the total NPVs do not differ very much, the pit geometries and total effort to produce the solutions do have considerable differences.

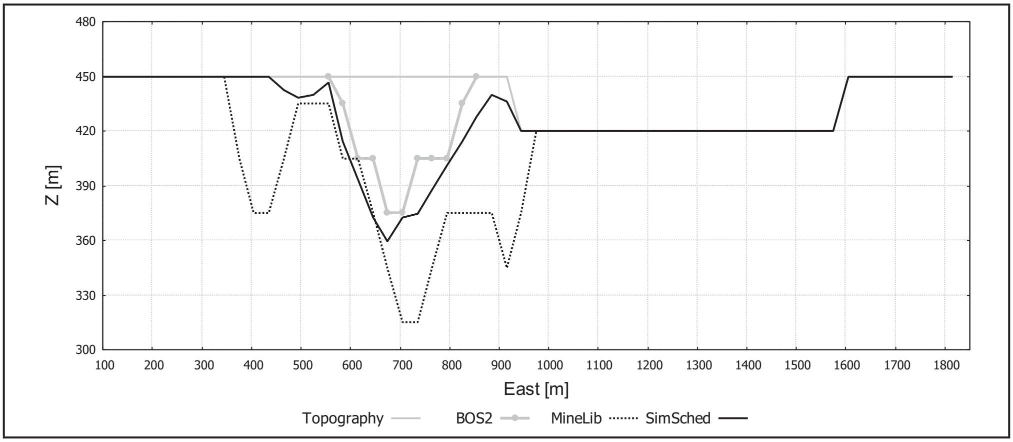

Figure 5. Marvin pit profile, Period 1

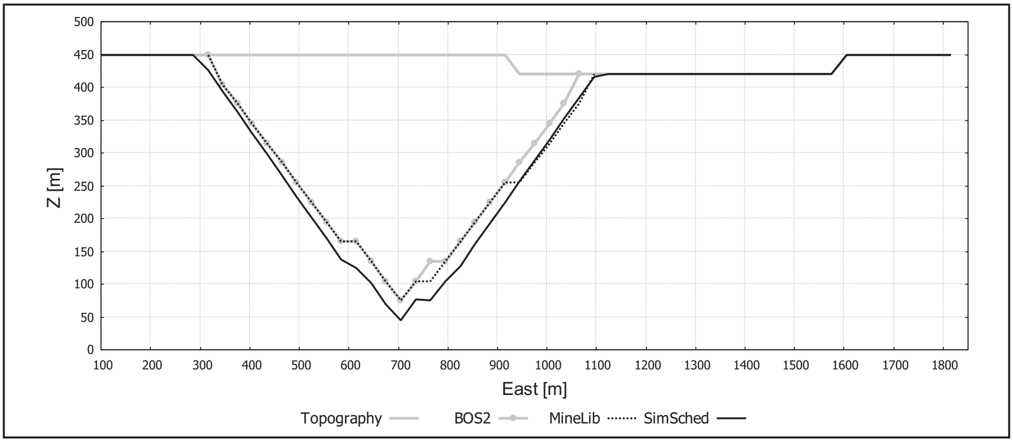

Figure 6. Marvin pit profile, Period 7

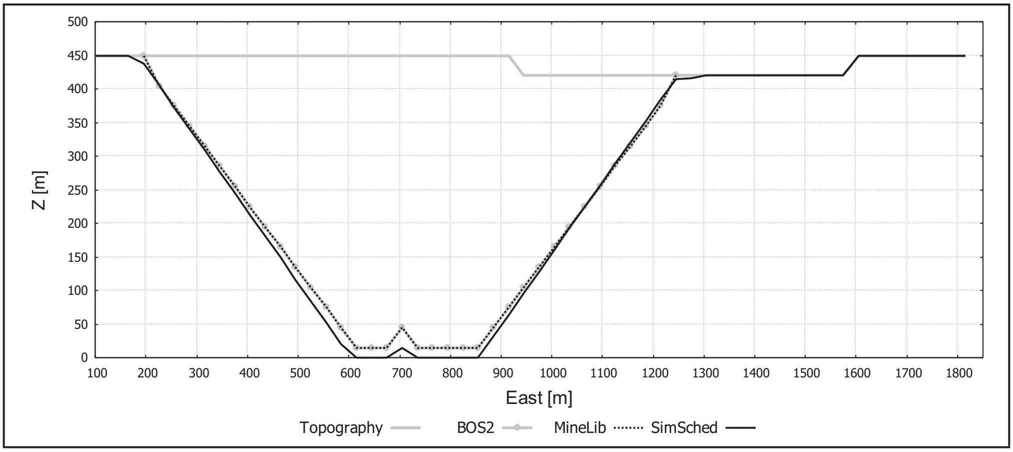

Figure 7. Marvin pit profile, Period 14

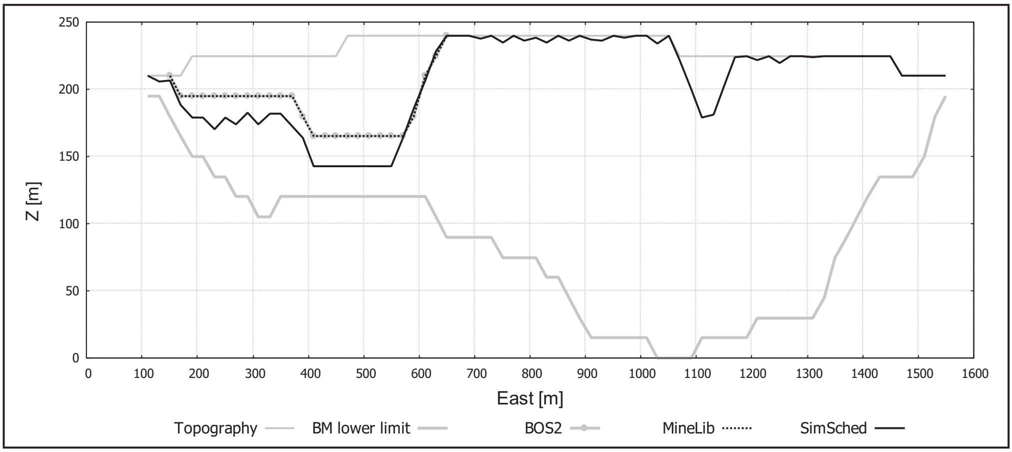

Figure 8. KD pit profile, Period 1

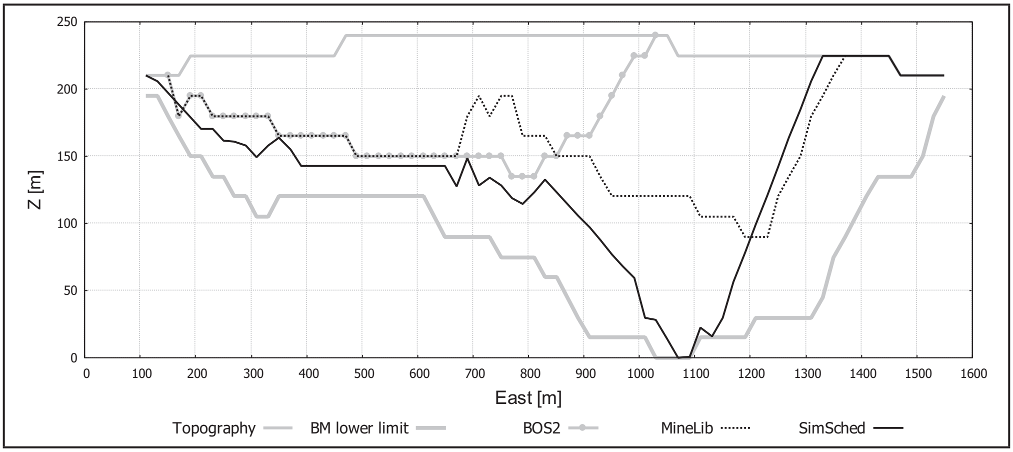

Figure 9. KD pit profile, Period 5

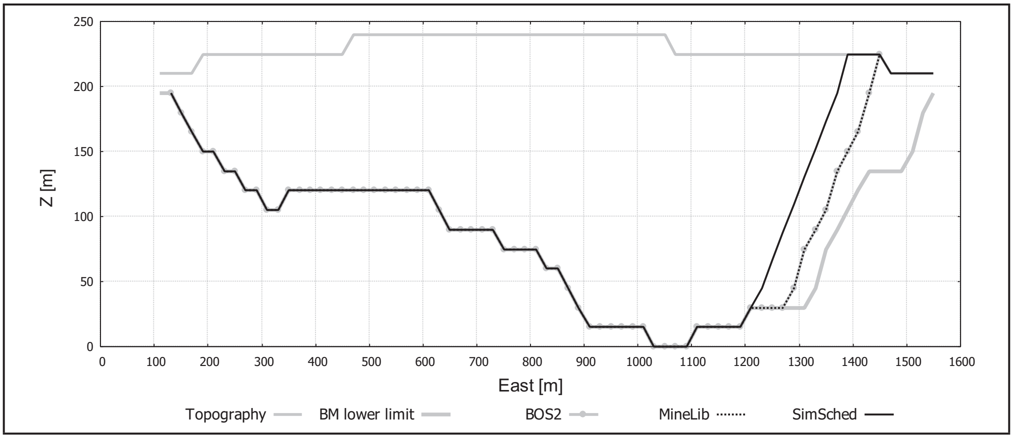

Figure 10. KD pit profile, Period 10

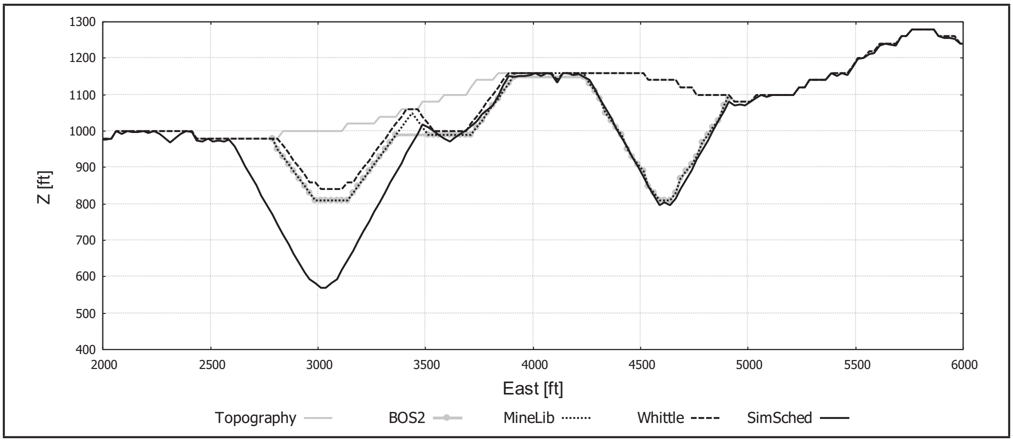

Figure 11. McLaughlin pit profile, Period 1

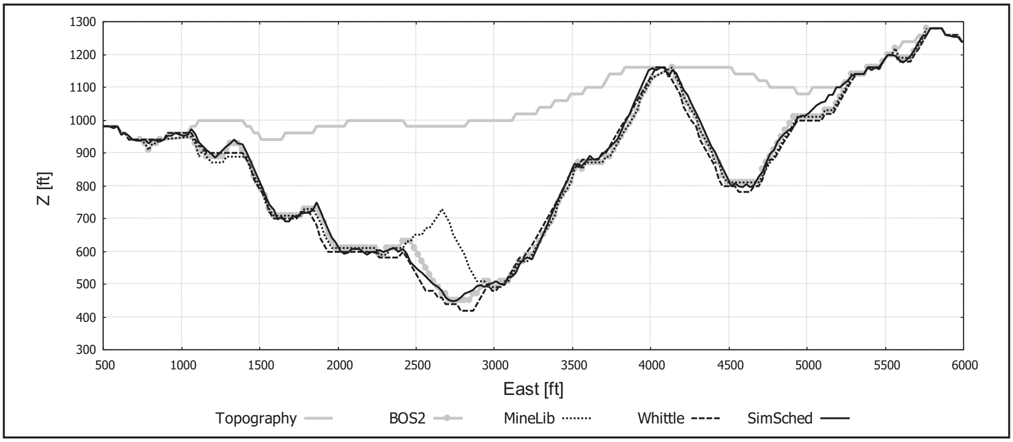

Figure 12. McLaughlin pit profile, Period 8

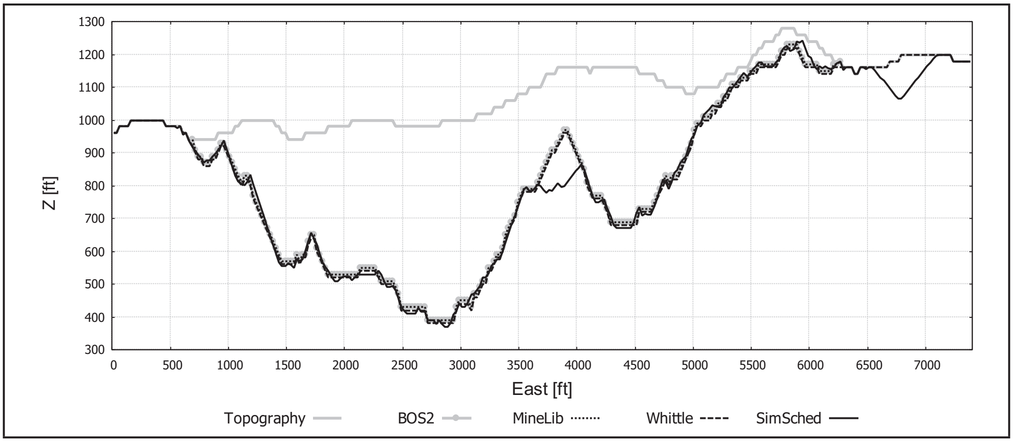

Figure 13. McLaughlin pit profile, Period 16

In terms of methodologies, the conventional method based on LG is somehow more complex, because it requires a lot of expertise for manual steps, like selecting pushbacks. This complexity translates to the fact that the overall process achieves similar performance (NPV-wise), but requires quite more effort than resorting to direct block scheduling models. Indeed, for the McLaughlin case, where we compare all the approximations, SimSched and BOS2 took in between 1.0 and 1.5 hours to generate the schedules presented in this paper in a single optimization run, but the schedule using Whittle required about 15 hours of a well-qualified planner for a series of tests, until the best solution was found. This scenario gets worse if more complex considerations like blending or geometric constraints have to be added. For example, the model from BOS2 can easily incorporate constraints on average grades for multiple processes or maximum pollutant content, and SimSched is also able to integrate geometric constraints in terms of minimum phase sizes.

In terms of the pit geometries, there seem to be very big differences, especially in the first time periods. This may be only a consequence of the case studies, but in fact these differences are the ones that introduce the NPV variations.

Overall, a main result is the fact that DBS is becoming a competitive alternative to optimize extraction in open pit mines, especially for complex cases or where multiple scenario analysis is required.

Relevant projections of the current work may include extending the analysis to more case studies. Unfortunately this is not directly possible for the problems available in MineLib. Other extensions are the inclusion of blending, other operational constraints, and geological and market uncertainty in the analysis.

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