An Integer Linear Programming Model for Optimizing Open Pit Ramp Design

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Open pit ramp design is a complex stage of mine planning that is often time consuming and highly dependent on the planner expertise. To help the mine planner to optimize this strategic step, we propose an integer linear programming model that starts from a given pushback at the block level, with ramp parameters and geotechnical constraints, and produces a new pushback that facilitates the ramp design. The aim of the optimization is to minimize the impact of the ramp design on the economic value, shape and tonnage of the original pushback.

Introduction

The problem of strategic open pit planning optimization is divided into several stages. In the first stage, the production plan optimization takes place, during which the mine planner determines the volumes, or pushbacks, to be scheduled for mining over time based on a valuated block model with the pushbacks defined at the block level. The second stage consists of the design of the pit phases, where the pushbacks are used as guides to plan the phases, including the ramps and operational spaces required for the material extraction. Finally, in the last stage, a mine plan is constructed by scheduling the benches of the designed phases.

The first and third steps are often supported by computational tools and optimization models. For example, the optimization of the production plan can be addressed using the final pit problem approach in which a set of blocks with maximum value is computed such that the requirement for safe slope angles are met. This problem does not take into account extraction capacities or blending constraints and, therefore, does not consider time of the operation. However, this approach is widely used and has proven to be very useful as shown by Lerchs and Grossman [16], who proposed an algorithm to solve the final pit problem and showed that the final pit can be parameterized to generate several nested pits or pushbacks. Indeed, this approach is commonly used in commercial software [7].

An alternative to the nested pits approach is the direct block scheduling, which was proposed by Johnson [11]. In this approach, blocks are scheduled in discretized time periods and the extraction capacities and blending constraints can be expressed for each time period. The objective of this approach is to maximize the time dependent discounted value of extracted blocks. Bienstock and Zuckerberg [1] proposed an algorithm based on the relaxation of capacity and blending constraints that significantly improves the time to obtain the continuous solution. Lambert et al. [12] presented a tutorial that explains various formulations for the constrained pit problem with extraction capacities constraints and fixed block destination. Literature is also abundant with respect to the direct block scheduling used in open pit scheduling heuristics [14], [15], [18]; Jelvez et al. [10] proposed a spatial aggregation heuristic to improve most of MineLib [9] solutions to the constrained pit problem.

On the other hand, for the second stage related to phase design, the automated or optimization tools available for the user are very few or non-existent. The ramp design is iterative, graphically done, time consuming and highly dependent on the planner´s expertise. To authors´ knowledge, there is no known publication that describes the whole open pit ramp design problem mathematically; there are publications that present the ramp design or part of the problem but without a mathematical model [17].
Sussman [19] developed a solution to the minimal path problem in a three dimensional space but without gradient constraint. Lee and Klette [13] proposed an algorithm that works in a three dimensional grid space of cube blocks. This algorithm solves some Euclidian Shortest Path (ESP) problems, for example, the shortest path inside polygons but also without gradient constraints.

Brazil et al. [2] aimed to solve the design of underground ramps by applying a theorem presented in Dubbins [6] in the three dimensional space with a fixed gradient. This theorem defines the shape of the admissible path of minimal length between two oriented points in the plane. An extended path method is used to optimize the underground ramps that can be lines or curves. The characteristics of this design are: the underground ramp curvature is fixed, both extremities of ramps and gradient are fixed, ramps must not go into specified zones and part of ramp can be above other part.
However, open pit ramp design is very different from underground design. For example, no part of ramp can be above other part, it is not necessarily known in advance where the ramps will end, the starting point of the ramp may be also part of optimization and the ramp curve may not need to be the same overall, among other constraints.
Therefore, an optimization model that aims to assist the planner in the design of the ramps for an open pit operation has been developed with an objective to maximise the economic value of the pushback that includes ramps and thus producing a new pushback, which is as close as possible to the original pushback.

Due to security reasons, there are generally more than one ramp in open pit mines. However, in this study, only one ramp was constructed to simplify the modelling. The more general problem of more than one ramp will be addressed in the forthcoming research.
The model works at the block model level and does not aim to produce a fully designed pushback, but rather to provide a guide to help the planner to better understand the issues related to the ramp design: such as, what are the best starting points for the ramps, what would be the impact if the design parameters (such as maximum ramp gradient, minimum ramp width, inter-ramp slope angle, overall slope angle) change or when the transportation costs change (for example, for different potential locations of waste dumps or stocks), all of that without needing to do the actual design of the mine.

Brief description of the proposed problem

Given a valuated block model and a pushback computed within this block model, a *pit boundary* of blocks around the limits of this pushback is considered. A new pushback (call *designed pushback*) is computed with its limits within the block boundary such that it contains enough space for the design of a ramp that goes from the top of the original pushback to the bottom of the *designed pushback*. Having this *pit boundary* guarantees that the resulting pushback is similar to the original one. The value of the *designed pushback* is computed by adding the values of its individual blocks that are extracted due to the ramp design and those that were inside the original pit (the *reduced pushback* is the part of the original pushback that exclude all the possible ramp blocks considering the given *pit boundary*).

In order to compute the *designed pushback*, at each bench a block that belongs to the ramp being designed is selected (we assume that it is the wall side of the ramp that corresponds to blocks in black in Figure 1a). This selection has to comply with several constraints. At a minimum, there are the ramp slope constraints, which define which blocks can be reached from a particular bench to the bench below. This is modelled using graph theory; it is assumed that there is an arc from Block b to Block b' if it is possible to draw a ramp from b to b'. The set of blocks in between these two is an elementary path that complies with the following definition.

$(b_1, b_2, \ldots, b_m)$ is a $level\ k\ elementary\ path$ if it is a directed path with no repeated blocks, and

$\forall\, j \in \{1,2,\ldots,m-2\}$, blocks b_j, b_{j+1} share exactly one face, and

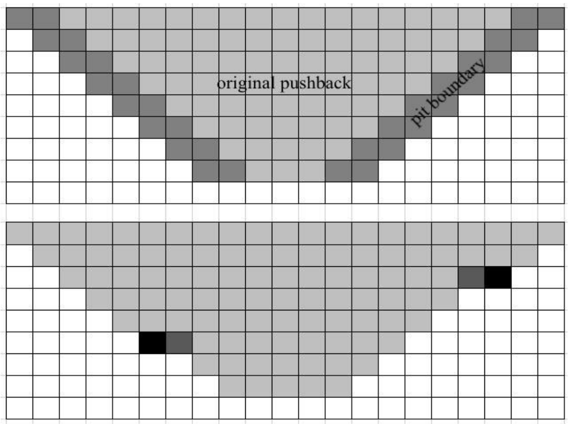
blocks b_{m-1}, b_m share exactly one edge, and

blocks $b_1, b_2, \ldots, b_{m-1}$ belong to bench k, and

block b_m belongs to bench $k-1$, and

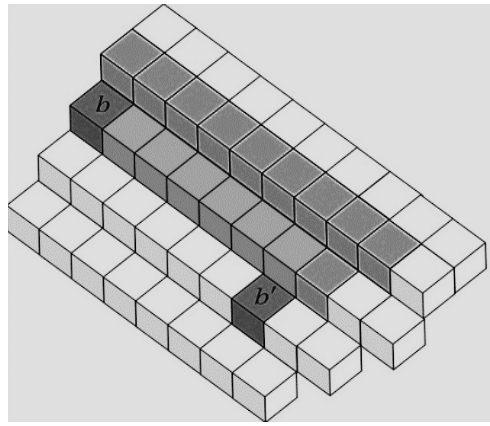
gradient of $(b_1, b_2, \ldots, b_m)$ is inferior or equal to the maximum ramp gradient.

Figure 1 illustrates these concepts. In Figure 1a (top), an original pushback and a possible *pit boundary* are shown. In Figure 1a (bottom), an example of *designed pushback* with ramps for the original pushback is shown. In this example, the ramps on wall side are in black and the minimal ramp width has the length of two blocks. The obscure grey blocks have been added to complete the ramp width. In Figure 1b, two blocks are selected and the blocks to be removed in order to give space for the ramp are shown. The *level k elementary path* is then computed as a shortest path between its two extremes on same level using the Dijkstra algorithm [3]. For this example, the minimal ramp width has the length of one block. Lineal optimization will choose which precomputed $level\ k\ elementary\ path$ to combine to generate the wall side part of the ramps.



(a) Top: Section view of a *pit boundary* in obscure grey comprised of two layers outside and zero layer inside an original pushback (in light grey).

Bottom: Section view of a *designed pushback* (in light grey) that includes ramps (in black and obscure grey).



(b) Isometric view of a *level k elementary path* with extremities *b* and *b'* (in black) and intermediate blocks (in light grey). Blocks in obscure grey have to be extracted to respect slope precedences.

Figure 1: Examples of *pit boundary, designed pushback* and *level k elementary path*

The model is completed by adding constraints that ensure the connectivity of the successive level paths and ramp width as well as constraints that prevent blocks to be extracted immediately below ramps, among others.
It can be seen in Figure 1b that the arcs from a block at bench k to the blocks reachable by a ramp at bench $k-1$ are crucial to the modelling of the problem as it is implicitly used to model not only the ramp gradient but also the possibility of using switchbacks, avoiding known obstacles, considering different gradients in different regions, among other options.

Optimization model

The nomenclature for the proposed mathematical modelling is as follows:

- B the block model
- K the maximum level at which the ramps can commence
- B_k the set of blocks of level $k, k \in \{0,1,\dots,K\}$, level 0 is lower level, K is ramp top level
- R the $reduced\ pushback$
- R_k $R \cap B_k$
- p_b the profit of block b
- p_{R_k} the sum of the profit of all blocks in the $reduced\ pit$ of level k
- I_k the set of index i of all k level elementary paths
- s_k^i the ith $level\ k\ elementary\ path$
- o_k^i the first block of s_k^i
- f_k^i the last block of s_k^i
- O_b the set of slope predecesors of block b; O_{R_k} the set of predecesors of R_k
- H_k^i the set of blocks of level k between s_k^i and R_k needed to be extracted to complete the ramp width
- L_k^i the set of blocks of level k between H_k^i and R_k needed to be extracted to prevent a railing between ramps and pit
- D_k^i the set of blocks of level $k-1$ immediately below the ramp blocks $s_k^i \cup H_k^i$
- Z_k^i the set of blocks of level k outside s_k^i (in the oposite direction of R_k) to avoid designing a pit between

ramps and slopes.

The variables of the problem are defined as follows:

$$y_b = \begin{cases} 1 & \text{if block } b \text{ is extracted,} \\ 0 & \text{otherwise.} \end{cases}$$

$$x_k^i = \begin{cases} 1 & \text{if all blocks of level k of } s_k^i \text{ are the wall side part of the ramp of level } k \\ & \text{and } f_k^i \text{ is the first block of the wall side part of the ramp of level } k-1, \\ 0 & \text{otherwise.} \end{cases}$$

Domain definition of variable *y* is *B*, domain definition of index *k* of variable *x* is *{1,2,...,K}*, domain definition of index *i* of variable *x* is I_k

Therefore, the Single Ramp Design Problem (*SRDP*), can be formulated as:

$$(SRDP) \quad \max \sum_{b \in B} p_b \, y_b \quad + p_{R_K} \quad + \sum_{k=1}^{K} \sum_{i \in I_k} p_{R_{k-1}} \, x_k^i \qquad (1)$$

$$s.t. \quad \sum_{i \in I_k \mid f_k^i = o_{k-1}^j} x_k^i \geq x_{k-1}^j \qquad (\forall k > 1, \forall j \in I_{k-1}) \qquad (2)$$

$$\sum_{i \in I_k} x_k^i \leq 1 \qquad (\forall k \geq 1) \qquad (3)$$

$$y_b \leq y_{b'} \qquad (\forall b \in R^C, \forall b' \in O_b \cap R^C) \qquad (4)$$

$$\sum_{i \in I_k} x_k^i \leq y_b \qquad \left(\forall k \geq 1, \forall b \in O_{R_{k-1}} \cap R^C\right) \qquad (5)$$

$$x_k^i \leq y_b \qquad (\forall k \geq 1, \forall b \in s_k^i \cup H_k^i \cup L_k^i) \qquad (6)$$

$$x_k^i + y_b \leq 1 \qquad (\forall k > 1, \forall b \in D_k^i \cup Z_k^i) \qquad (7)$$

$$\sum_{i \in I_k} x_k^i \geq y_b \qquad (\forall k \geq 1, \forall b \in B_{k-1} \setminus R_{k-1}) \qquad (8)$$

$$x_k^i = 0 \qquad (\forall k < K, \forall i \in I_k \mid \{s_{k+1}^j \mid f_{k+1}^j = o_k^i\} = \emptyset) \qquad (9)$$

$$y_i = 0 \qquad (\forall b \in R) \qquad (10)$$

The objective function (1) maximizes the overall profit of the ramp design and corresponds to the profit of all extracted blocks. As all *reduced pushback* blocks are considered as not extracted by our model but are extracted in the reality, adding a ramp path between level *k* and level *k - 1* will add the profit of all blocks of R_{k-1}.
Constraint (2) ensures the connectivity between ramp paths. Constraint (3) states that there is at most one ramp per level. Constraint (4) and constraint (5) prevent the extraction of any block for which all the slope predecessors have not been previously extracted. Constraint (6) ensures that for each chosen path, all blocks in the *level k elementary path*, all blocks needed to complete the ramp width and all blocks between ramps and the *reduced pushback* of level *k* are extracted. Constraint (7) prevents the extraction of blocks immediately below ramp blocks and blocks between the ramps and the slope. Constraint (8) guaranties that there is a *level k elementary path* on level above extracted blocks and so prevents the extraction of blocks below the down extremity of bottom ramp. Constraint (9) prevents a no connected path from being an eligible path. Constraint (10) prevents the extraction of all *reduced pushback* blocks as they are already taken into account in the objective and in constraint (5) for slope precedence.

Numerical application

To generate the entry data sets and implement the model, the MineLink library [4] developed at Delphos Mine Planning Laboratory at Universidad de Chile was used.

The *SRDP* model was applied to two block models. The first, Marvin30, was based on Marvin [7] and consisted of 53,271 blocks of size 30mx30mx30m. The second Marvin15 was obtained by splitting blocks from the first model and was composed of 426,168 cubic blocks with edges of 15 meters.
The original pushback in both cases was a final pit that was computed with precedence slope angle of 47°, but with 20 levels of precedence for Marvin15 and 10 levels for Marvin30. The parameters in Table 1 were used to compute the economic value of the blocks. The final pit values are presented in Table 2. The results of the ramp designs are presented in Table 3.

Table 1: Cost and Price parameters for the valorization of blocks

Parameter	Unit	Value
Mine Cost	US$ / ton	0.9
Process Cost	US$ / ton	4.0
Au Recovery	%	60
Cu Recovery	%	88
Au Price	US$ / Oz	344
Cu Price	US$ / lb	0.91
Selling Cost Au	US$ / Oz	5.74
Selling Cost Cu	US$ / lb	0.33

Table 2: Original pushbacks values and tonnage for Marvin30 and Marvin15

Block model name	Final pit value MUSD	Tonnage MT
Marvin30	1,405	543.5
Marvin15	1,442	516.4

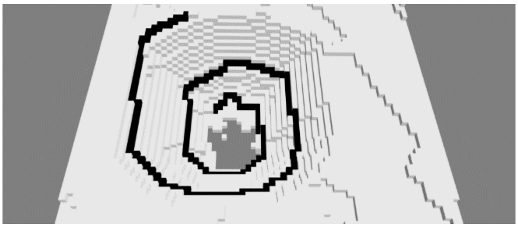
Ramp width of 30 meters and maximum ramp gradient of 10% are parameters used for all the ramp design computes. For Marvin30, a *pit boundary* consisting of two external layers (outside the original pushback) was considered and, therefore, the *designed pushback* contained the original pushback if ramps reached the bottom level of the original *pushback*. For Marvin15, a *pit boundary* consisting of one internal layer and one external layer was considered, which means that the ramp could go outside the original pushback.
Three cases were considered. For Marvin30, there are *M30_fixed_start* and *M30_free_start*, which refer to the initial block for the ramp being fixed or not, respectively. For Marvin15, there was only *M15_fixed_start*, i.e. the starting block was fixed. All the elementary paths generated for the three cases were in the counterclockwise directions. Gurobi 6.5 [8] was used to solve the three case studies with a relative optimality gap of 0.5%.

Table 3: *SRDP* model ramp design results and tonnages of the *designed pushback*

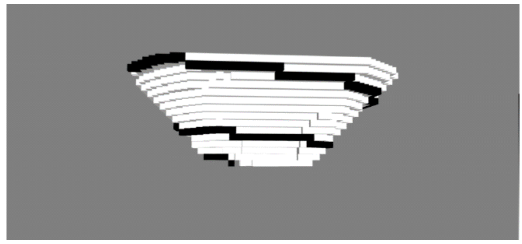
Instance name	# elementary paths	Objective value MUSD	Gap %	Tonnage MT
M30_fixed_start	20,310	1,431	0.35	572.2
M30_free_start	20,620	1,432	0.49	567.6
M15_fixed_start	69,990	1,440	0.46	514.2

It was observed that, for Marvin30, the economic values of the designed pushbacks were higher than those for the original pushback (which corresponds to a final pit). This was due to using an inter-ramp slope angle of 50° and an overall slope angle of 47° for the slope precedence, which resulted to be less restrictive and the solutions were able to recover a slightly higher amount of mineral that leads to a better value (up to + 1.9%).
On the other hand, in the case of Marvin15, the block boundary was tighter (one internal and one external layer, plus blocks of size 15mx15mx15m) and, therefore, the differences between the values (0.1%) and tonnages (0.4%) of the original and designed pushbacks were minimal.
Tonnage of ramp design for Marvin30 experiments were higher as compare with the initial pushback tonnage (5.3% and 4.4%). This was expected as no internal layer in the boundary forced the pushback to grow.
Finally, for Marvin15, the model attained a slightly higher economic value (0.6%) with lower tonnage, which was due to the increased precision of using smaller blocks.
Figures 2a and 3a present the ramps in black in result pit for the both instances of Marvin30. Figures 2b and 3b present the *designed pushback* that include *reduced pushback* blocks that are aggregated by a post processing of *SRDP* results,

with also ramps in black. For Figures 2a and 3a, all blocks of the *designed pushback* but ramps have been taken away to allow to see the ramp paths.

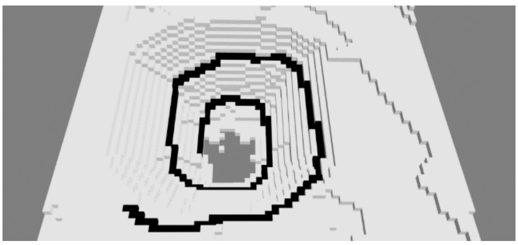


(a) Isometric view of ramps in result pushback

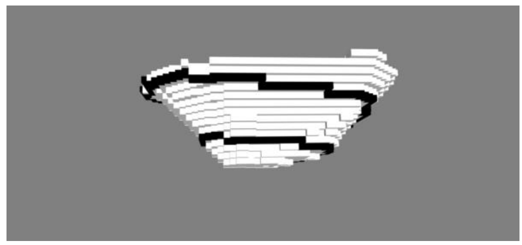


(b) Lateral west view of ramp design

Figure 2: Results for *M30_fixed_start* (ramp in black)



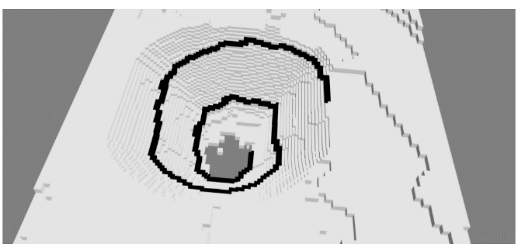
(a) Isometric view of ramps in result pushback



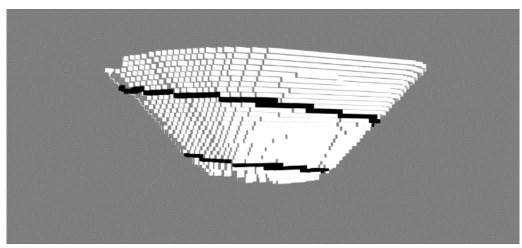
(b) Lateral east view of Ramp design

Figure 3: Results for *M30_free_start* (ramp in black)

Figure 4 (a) shows the ramps in black in the result pit of *Marvin15_start_fixed*. Figure 4 (b) presents the *designed pushback* that was the result of ramp design computed for *Marvin15_start_fixed*. The graphical representation of the results, as shown in Figures 2, 3 and 4, were generated with the academic software DOPPLER [3].



(a) Isometric view of ramps in result pushback



(b) Lateral west view of Ramp design

Figure 4: Results for *M15_fixed_start* (ramp in black)

Conclusions

We presented a linear optimization model that aims to assist the planner in the design of the ramps for an open pit operation with an objective to maximize the value of the pushback that includes ramps. The model was created based on an original pushback and generated a *designed pushback*, which aimed to provide a guide to the planner to analyze different alternatives and scenarios quickly without the need to perform an actual design.

The model was implemented and applied to three case studies, showing consistent results obtained with the branch and bound. The selection of the pit boundary impacts the extracted tonnage of the *designed pushback* and as the number of external layers included in the analysis grows, the tonnage increases.

This initial analysis provides an excellent basis to extend the model to consider multiple ramp design and dynamic cases with several pushbacks or phases. In addition, as the model considers the block values, the impact of the location of waste dumps or stocks and variation in truck fleet could be studied with this model.

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References

[1] Bienstock, D. and Zuckerberg, M., “Solving LP relaxations of large-scale precedence constrained problems”. In IPCO, pp. 1–14, 2010.

[2] Brazil, K., Thomas, D.A., Weng, J.F., Rubinstein, J.H. and Lee, D.H., “Cost Optimisation for Underground Mining Networks”, Kluwer Academic Publishers, 2003.

[3] Delphos, DOPPLER, Delphos Open Pit Planner, http://delphos.dmi.uchile.cl, 2015.

[4] Delphos, M. P. L. Minelink library, http://delphos.dmi.uchile.cl, 2013.

[5] Dijkstra, E., W., “A short introduction to the art of programming”, pp. 67-73, 1971.

[6] Dubins, L.E., “On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents”, American Journal of Math. Vol. 79, n° 3, pp. 497-516, 1957.

[7] Gemcom WhittleTM Strategic Mine Planning software, http://www.gemcomsoftware.com/products/whittle, 2011.

[8] Gurobi Optimization, Gurobi optimizer reference manual. http://www.gurobi.com, 2016.

[9] Espinoza, D., Goycoolea, M., Moreno, E., and Newman, A., “Minelib: a library of open pit mining problems”, Annals of Operations Research, Vol. 206, pp. 93–114, 2013.

[10] Jelvez, E., Morales, N., Nancel-Penard, P., Peypouquet, J. and Reyes, P., “Aggregation heuristic for the open-pit block scheduling problem”, European Journal Of Operational Research, Vol. 249, n°3, pp. 1169-1177, 2016.

[11] Johnson, T.B., “Optimum open-pit mine production scheduling”, Phd thesis, Operations Research Department, University of California, Berkeley, 1968.

[12] Lambert, W. and Brickey, A. and Newman, A. and Eurek, K., “Open-Pit Block-Sequencing Formulations: A Tutorial”, Interfaces, Vol. 44, n°2, pp. 127-142, 2014.

[13] Lee, F., Klette, R., “Rubberband Algorithms for Solving Various 2D or 3D Shortest Path Problems”, International Conference on Computing: Theory and Applications, IEEE, pp. 9-19, 2007.

[14] Lamghari, A., Dimitrakopoulos, R. and Ferland, J. A., “A hybrid method based on linear programming and variable neighborhood descent for scheduling production in open-pit mines”, Journal of Global Optimization, Vol. 63, pp. 555-582, 2014.

[15] Liu, S. Q. and Kozan, E., “New graph-based algorithm to efficiently solve large scale open pit mining optimization problems”, Expert Systems With Applications, Vol. 43, pp. 59-65, 2016.

[16] Lerchs, H. and Grossman, H.C., “Optimal design of open-pit mines”, Transactions C.I.M., Vol. 58, pp. 47-54, 1965.

[17] Parra, A., Morales N., Vallejos, J., Nguyen, P. M. V., “Open pit mine planning considering geomechanical fundamentals”, International Journal of Mining, Reclamation and Environment, pp. 1-18, 2017.

[18] Samavati, M., Essam, D., Nehring, M. and Sarker , R., “A local branching heuristic for the open pit mine production scheduling problem”, European Journal of Operational Research, Vol. 257, pp. 261-271, 2017.

[19] Sussman, H.J., “Shortest 3-dimensional paths with prescribed curvature bound”, Proc. 34th Conf. on Decision and Control, IEEE, pp. 3306-3312, 1995.