A New Methodology for the Automated Pushback Selection

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ABSTRACT

The design of pushbacks is essential to long-term open pit mine scheduling because it partitions the pit space into individual units, controlling ore and waste production. In this paper, we propose a new methodology for the pushback selection procedure, which consists of characterizing the potential pushbacks based on the comprehensive family of nested pits and selecting those pushbacks that meet set criteria, for instance, bounded ore and waste. An advantage of this method is the possibility to automate the pushback selection methodology, applying well-defined criteria for the pushback selection and reducing the time employed in the planning task.

INTRODUCTION

The design of pushbacks is essential to long-term open pit mine scheduling because it partitions the pit space into individual units, controlling ore and waste production. The pushbacks are used as a guide for the subsequent block scheduling stage, defining where the extraction process begins and where it stops. In addition, pushbacks ensure safe pit walls, assist in meeting the ore production requirement and provide a minimum operational width to accommodate mining equipment and different access to the mine, among other activities.

Traditionally, the pushback selection is done manually by expert mine planning engineers, based on the nested pits obtained using the methodology developed by Lerchs and Grossmann (1965). From the total number of generated nested pits, a selected number is used to define the pushbacks, based upon selected criterion (or criteria), for instance, minimum operational width that must be maintained. However, this procedure has important limitations: (i) it does not guarantee that the ore and waste tonnages are uniformly distributed between the pushbacks, which could affect the quality of the scheduling stage (also known as "gap problem"); (ii) the in-situ grade uncertainty is not taken into account; and (iii) generally, the selection of pushbacks is a subjective decision of a mine planning engineer. A complete reference for this topic can be found in Meagher et al. (2014), where a review of methods are examined in order to produce pushback designs, particularly, how they can tackle the gap problem.

In this paper, we propose a new methodology for the pushback selection procedure, which consists of characterizing the potential pushbacks based on the comprehensive family of nested pits and selecting those pushbacks that meet certain specified conditions, for instance, bounded ore and waste.

This paper provides details of the new methodology followed by the illustration of its performance using a gold deposit as case study.

METHODOLOGY

Assumptions and notations

The developed methodology is based on the following assumptions and notations:

1) A set of blocks named B.
2) This set B represents the final pit of the deposit and the blocks are denoted with letters b and b', without loss of generality.
3) Nested pits are generated using an economic block model and a metal price parameterization. Lerchs and Grossmann (1965) observed that obtaining the nested pits is equivalent to finding the maximum closure of a graph and proposed a customized algorithm (for modern algorithms, see Chandran and Hochbaum, 2009).
4) The generation of nested pits has a great impact on finding feasible solutions. Therefore, it is

necessary to define a sequence of revenue factors $RF_1 < RF_2 < \cdots < RF_N$, where $0 < RF_i \leq 1$ and which are used to scale the metal price. The block value is obtained as follows (Equation 1):

$$v_b^i = \begin{cases} [(p \cdot RF_i - c_s) \cdot g_b \cdot R - c_p] \cdot ton_b - c_m \cdot ton_b, & if\ (p \cdot RF_i - c_s) \cdot g_b \cdot R \geq c_p + c_m \\ -c_m \cdot ton_b & , \quad otherwise. \end{cases} \tag{1}$$

where v_b^i represents the value of block b when the metal price is scaled by the i^{th} revenue factor (each revenue factor produces a nested pit), p is the metal price, c_s is the selling cost, g_b is the ore grade of block b, R represents the metallurgical recovery, c_p and c_m the processing and mining cost, respectively, and ton_b is the total tonnage of block b. All factors must be in the correct units.

5) Solving Equation 1 leads to the final pit type problems (FPP) obtaining a family of nested pit (Equations 2 through 4):

$$(FPP_i) \quad max \sum_{b \in B} v_b^i \cdot x_b \tag{2}$$

$$x_a \leq x_b \qquad , \forall\ b \in B, b' \in PREC(b) \tag{3}$$

$$x_b \in \{0,1\} \qquad , \forall\ b \in B. \tag{4}$$

where x_b is a binary variable for final pit (in this case, for intermediate pit) selection and $PREC\,(b)$ represents the set of precedence of block b.

6) Let N be the number of nested pits generated by metal price parameterization and let P_i be the i^{th} nested pit, with $i = 1, \dots, N$. Let us define a dummy pit as one without blocks (empty set), denoted as P_0. The property of nested pits is as follows (Equation 5):

$$P_0 \subset P_1 \subset \cdots \subset P_N \tag{5}$$

7) Based on the set of assumptions, the pushbacks are defined as the set of blocks within the difference between two pits (Equation 6):

$$Push\,(i,j) = P_i - P_j \qquad , \forall\ i = 1, \dots, N; j = 0, \dots, N-1. \tag{6}$$

8) In addition, given N nested pits:

i) the set of pushback will be denoted by X.
ii) the total number of pushbacks is $\frac{N(N+1)}{2}$.
iii) there exist 2^{N-1} different ways to select a partition of the final pit based on pushbacks.

It is impossible to test all combinations (enumerative schema), considering the exponential behaviour of (iii).

Figure 1 shows a simplified procedure: a set of nested pit is calculated and, based on selected criteria, for instance, bounded ore and rock tonnages, the best combination of pushbacks is selected.

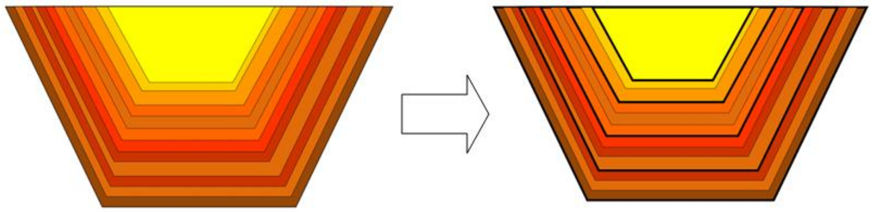

Figure 1: Nested pits generated by a parameterization of the metal price. Some of these pits are selected as pushbacks.

9) For each pushback, we define $rton_{ij}$ as total tonnage (rock tonnage) of pushback $Push(i,j)$. Similarly $oton_{ij}$ represents ore tonnage of pushback $Push(i,j)$. The desired rock tonnage and ore tonnage into each pushback are given by upper and lower limits and they are denoted respectively for: (i) rock tonnage in pushback $Push(i,j)$, RT_{ij}^{+} and RT_{ij}^{-}; and for (ii) ore tonnage in pushback $Push(i,j)$, OT_{ij}^{+} and OT_{ij}^{-}.

10) To automate the pushback selection, it is necessary to characterize the family of potential pushbacks. A set of predecessor pushbacks is defined at a given one $Push(i_o, j_o)$, where $i_o = 2, \dots, N$ and $j_o = 1, \dots, i_o - 1$ is denoted by (Equation 7):

$$PREC_{i_o j_o} = \{Push(i,j) \in X : i = j_o, j = 0, \dots, j_o - 1\} \qquad (7)$$

11) Similarly, the set of successor pushbacks at a given one $Push_{i_o j_o}$, where $i_o = 1, \dots, N-1$ and $j_o = 0, \dots, i_o - 1$ is (Equation 8):

$$SUC_{i_o j_o} = \{Push(i,j) \in X : i = i_o + 1, \dots, N, j = i_o\} \qquad (8)$$

Model creation

In this section, the details of the IP model that automate the pushback selection are provided.

Variables

The variables related to the decision of whether to select or not a given pushback are (Equation 9):

$$x_{ij} = \begin{cases} 1, & \text{if } Push(i,j) \text{ allows to pushback selection,} \\ 0, & \text{otherwise.} \end{cases} \qquad (9)$$

Objective

There exist a number of alternatives in order to define the objective function. For example,

- minimization/maximization of number of pushback

- minimization of the tonnage differences among selected pushbacks (gap problem)
- maximization of ore tonnage within each pushback

Constraints

The criteria (constraint) for pushback selection are defined according to the following:

- **Pushback predecessors:** in order to select a given pushback, one and only one of their predecessors must be selected according to Equations 10 and 11:

$$x_{ij} \leq \sum_{Push\,(u,v) \in PREC_{ij}} x_{uv} \qquad , \forall\, i = 2, \dots, N; j = 1, \dots, i-1 \tag{10}$$

$$\sum_{Push\,(u,v) \in PREC_{ij}} x_{uv} \leq 1 \qquad , \forall\, i = 2, \dots, N; j = 1, \dots, i-1 \tag{11}$$

- **Pushback successors:** one and only one of the successors must be selected to include a given pushback according to Equations 12 and 13:

$$x_{ij} \leq \sum_{Push\,(u,v) \in SUC_{ij}} x_{uv} \qquad , \forall\, i = 1, \dots, N-1; j = 0, \dots, i-1 \tag{12}$$

$$\sum_{Push\,(u,v) \in SUC_{ij}} x_{uv} \leq 1 \qquad , \forall\, i = 1, \dots, N-1; j = 0, \dots, i-1 \tag{13}$$

- **Partition of final pit:** the pushback selection must be a partition of final pit. That is, the initial and the final pushbacks must be extracted (Equations 14 and 15).

$$\sum_{i=1}^{N} x_{i0} = 1 \tag{14}$$

$$\sum_{j=0}^{N-1} x_{Nj} = 1 \tag{15}$$

- **Capacity:** the quantity of ore and rock (waste + ore) tonnages should be bounded (Equations 16 and 17):

$$OT_{ij}^{-} \cdot x_{ij} \leq oton_{ij} \cdot x_{ij} \leq OT_{ij}^{+} \qquad , \forall\, i = 1, \dots, N; j = 0, \dots, i-1 \tag{16}$$

$$RT_{ij}^{-} \cdot x_{ij} \leq rton_{ij} \cdot x_{ij} \leq RT_{ij}^{+} \qquad , \forall\, i = 1, \dots, N; j = 0, \dots, i-1 \tag{17}$$

CASE STUDY

The defined methodology was applied to the McLaughlin gold deposit, which was located in the northern Coast Ranges of California. This deposit was mined from 1985 until 1996, but gold processing continued through 2002. It was a world-class gold orebody, and one of the world's finest examples of a hot spring-type epithermal precious metals system. The dataset was obtained from the Minelib library (Espinoza et al., 2012). The parameters used for the experiment are shown in the Table 1.

In the first step, the nested pits were calculated. Considering a family of revenue factors as $RF_i = i/200$, for $i = 1, \ldots, 200$, and solving repeatedly (FPP_i), a set of nested pits was found (algorithms exist to solve this mathematical problem, see for instance, Hochbaum (2008)). The objective chosen was the minimization of the number of pushback, subject to upper and lower bound of ore and rock tonnage.

Table 1 Parameters for the experiments related to nested pits

Technical and Economical Parameters	**Value**	**Model Parameters**	**Value**
Slope angle (°)	45	Number of pits	200
Recovery	0.76	OT^-	20,000,000
Gold price ($/oz)	1,100	OT^+	40,000,000
Mining cost ($/ton)	1.5	RT^-	0
Processing cost ($/ton)	8.2	RT^+	60,000,000
Selling cost ($/oz)	100		

RESULTS AND DISCUSSION

In the first stage, 200 nested pits were generated. Figure 2 shows both plant and section views from the obtained results, which proves the existence of two well defined areas within the deposit where the extraction will be carry out and which are separated in the middle. However, some areas (corners) are not operational; therefore, it is necessary to manipulate them manually, an acceptable procedure in the selection of the nested pits. In the Figure 4 (left), the pit by pit graph shows the tonnages (ore and waste) for each pit and the cumulative undiscounting value. Traditionally, this graph is used to identify candidates for pushback.

Figure 3 presents the model´s geometrical results that define pushbacks from nested pits, by using the parameters defined in previous section. Six pushbacks were obtained; the ore and waste tonnages for each are shown in the Figure 4 (right). The average ore tonnage per pushback was 32.3 [MTon] with a standard deviation (SD) of 6.8 [MTon]. The average rock tonnage per pushback was 41.8 [MTon] with SD of 11.1 [MTon]. An important aspect of this methodology is the time required for arriving at the final solution: the nested pit generation (200) was computed in 30 mins, while the automated pushback selection was computed in 5 mins.

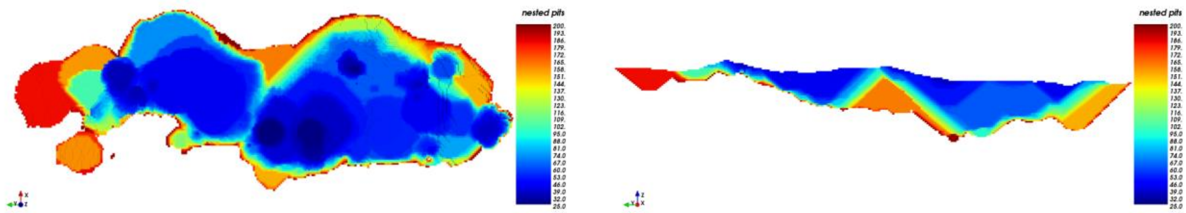

Figure 2: Plant view (left) and section view (right) of nested pits map, McLaughlin deposit.

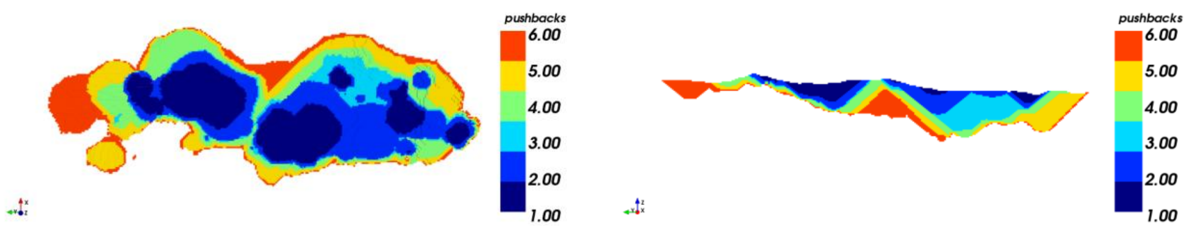

Figure 3: Plant view (left) and section view (right) of pushback selection from McLaughlin mine.

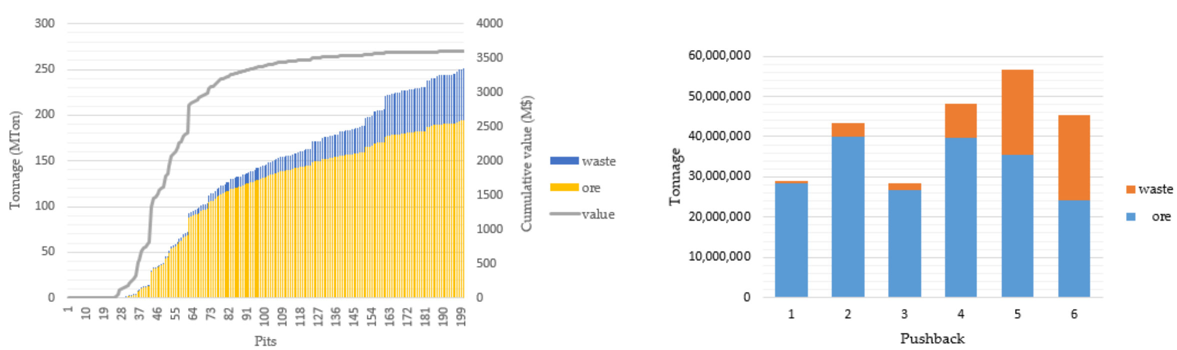

Figure 4: Pit by pit graph (left) for 200 nested pits. Tonnage graph (right) of pushback selection from McLaughlin mine.

CONCLUSION

In this paper a new methodology was proposed for solving the problem of pushback selection from a set of nested pits. The model works by properly characterizing the pushbacks and selecting important criteria that must be respected, such as, bounded ore and total tonnages. The results show good performance in terms of implementation and times required for computation of the final pushbacks; it took 5 mins to generate a pushback selection. This represents a new alternative to be incorporated within the traditional way of pushback selection, because the computing speed allows the analysis of various scenarios according to the chosen criteria with minimum time.

Other criteria can be considered, such as minimum operational width between pushbacks; simplifying the time required for the nested pit generation stage; reducing the number of variables when the revenue factors get identical nested pits that could be implemented within the proposed computational method.

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